Identification

Brady Neal

causalcourse.com

The magic of randomized experiments

Frontdoor adjustment

Pearl's *do*-calculus

Determining identifiability from the graph

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Determining identifiability from the graph

Randomized experiments are magic.

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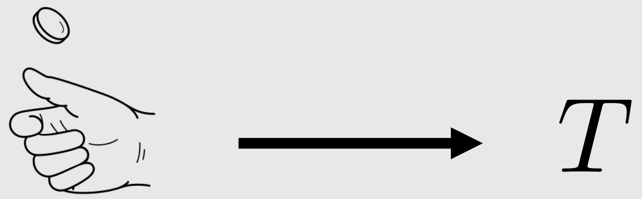
No unobserved confounding

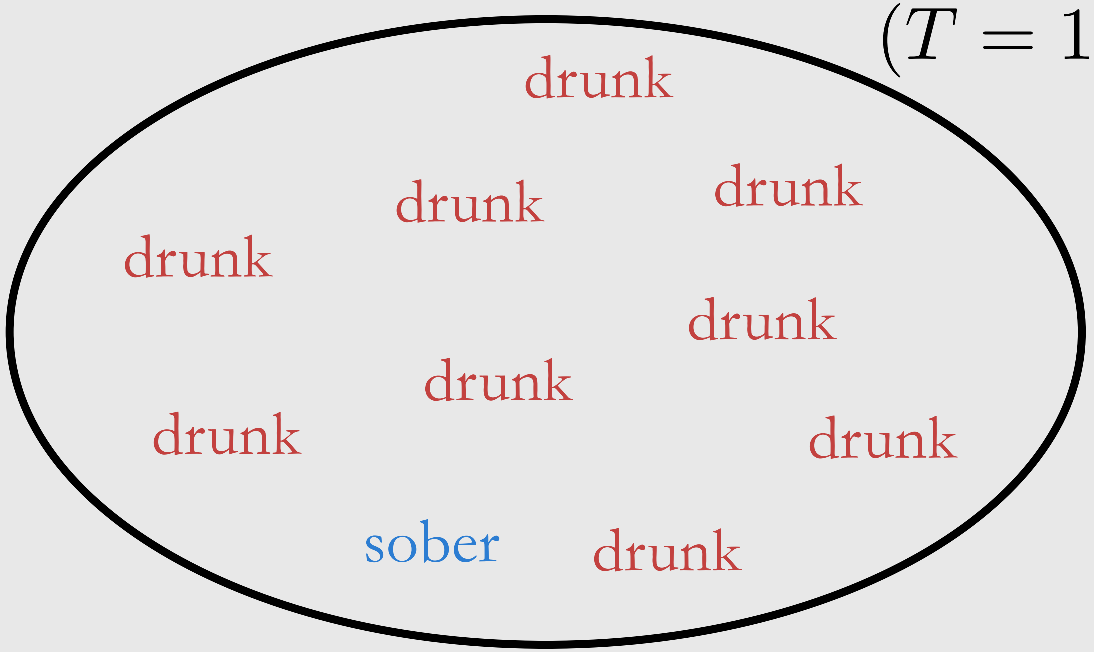
Randomized control trial (RCT)



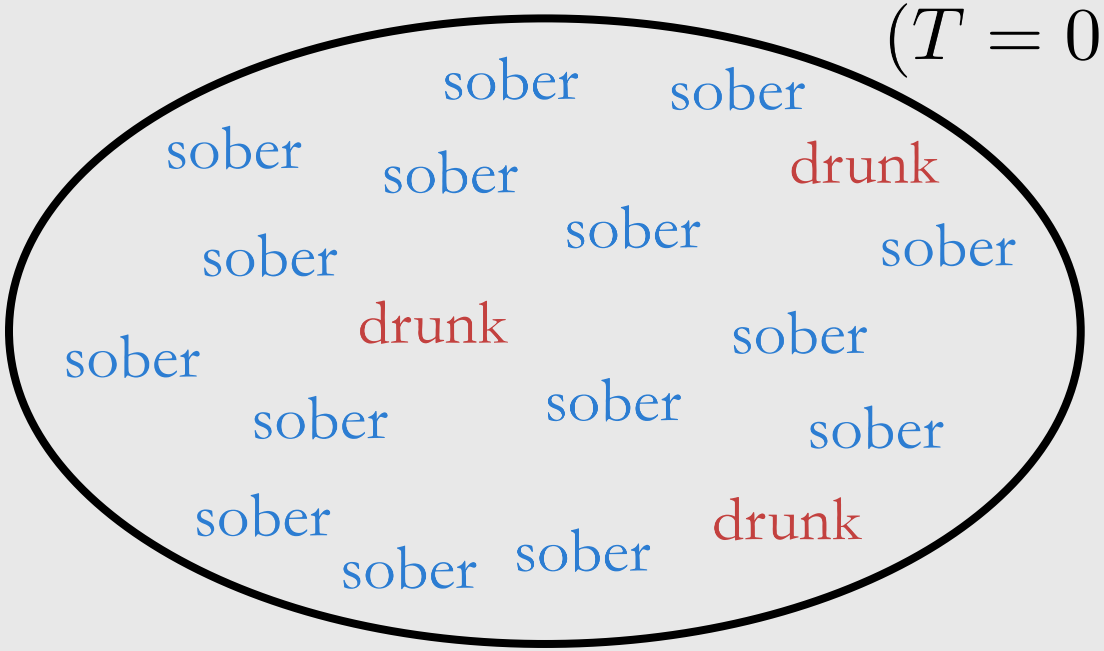
Randomized control trial (RCT)

$\longrightarrow T$

Went to sleep **with shoes** on $(T = 1)$



Went to sleep **without shoes** on $(T = 0)$



Randomized control trial (RCT)

 T

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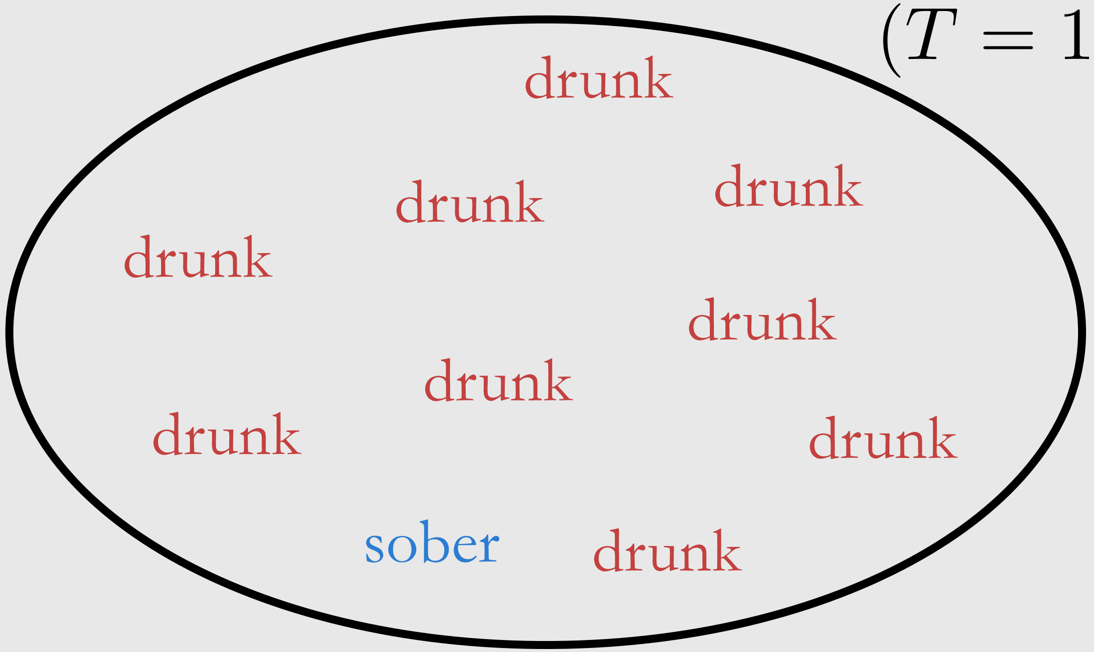

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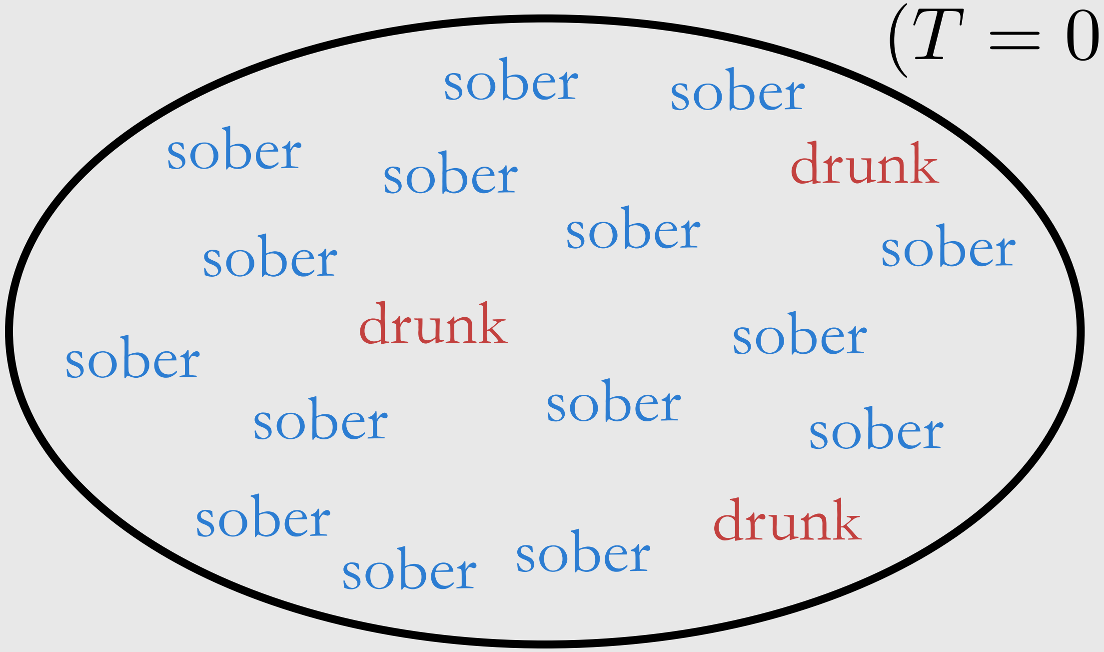

Randomized control trial (RCT)

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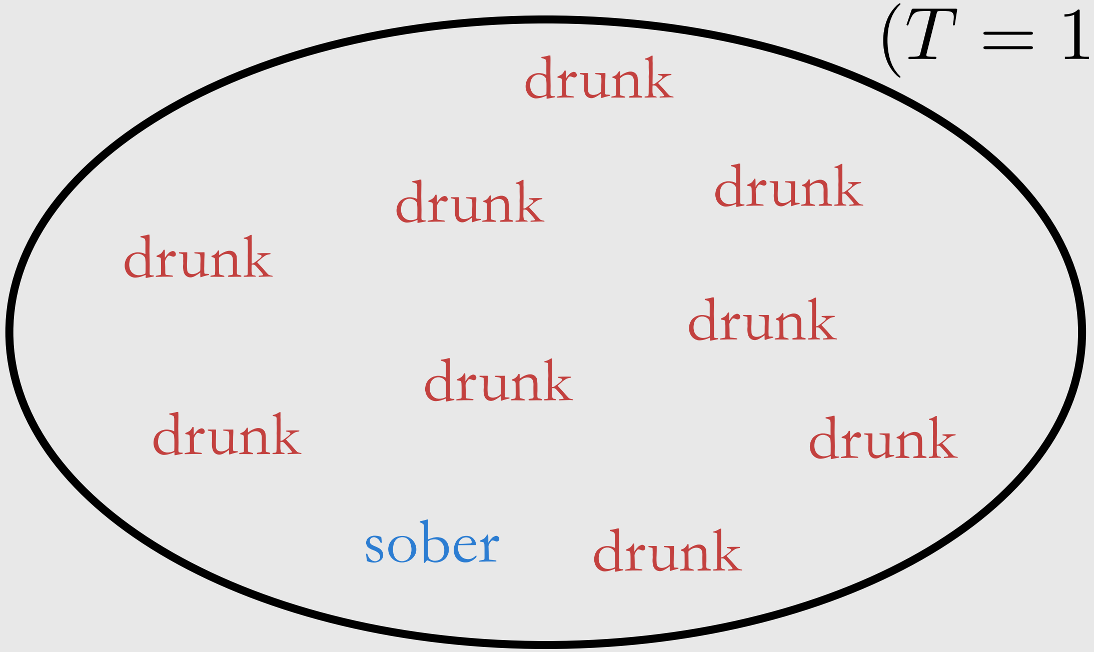
$(T = 1)$



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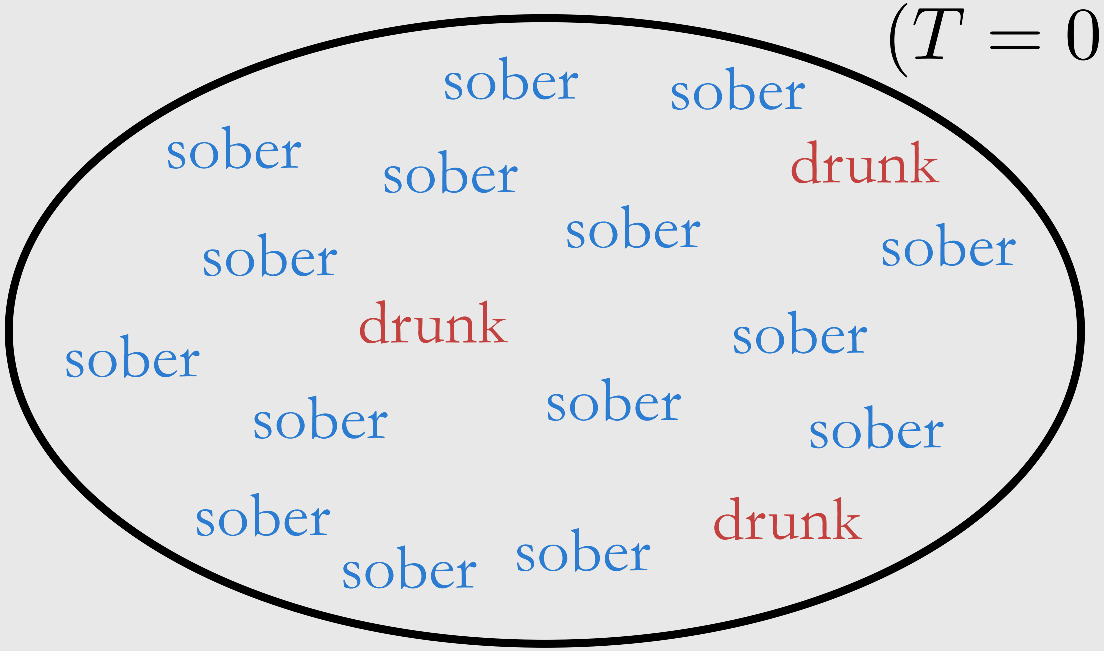

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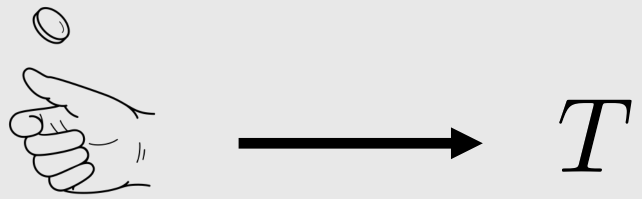
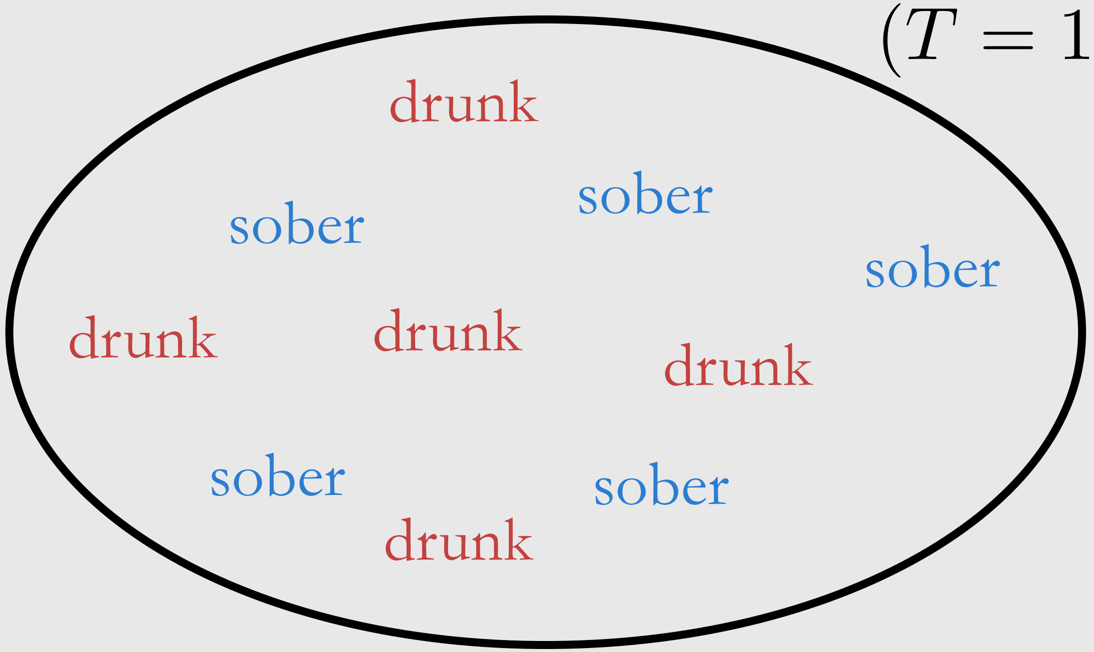


Slept **with shoes** on ($T = 1$)



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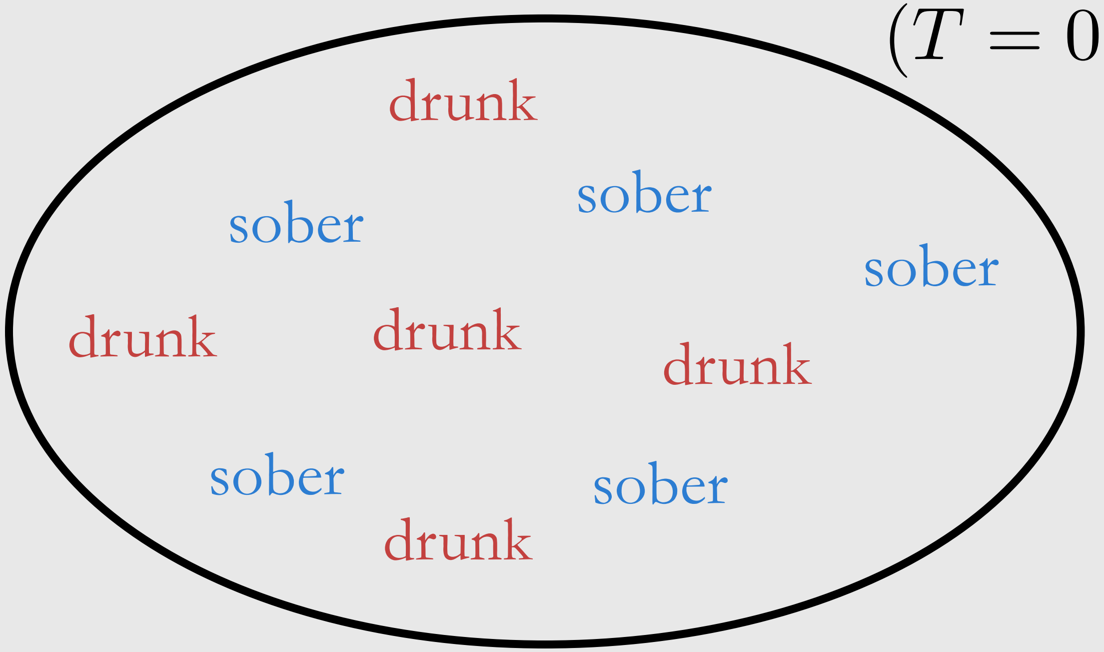

Few different perspectives on the magic

Comparability and covariate balance

Exchangeability

No backdoor paths

Comparability and covariate balance: intuition

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Treatment and control groups are the same in all aspects except treatment

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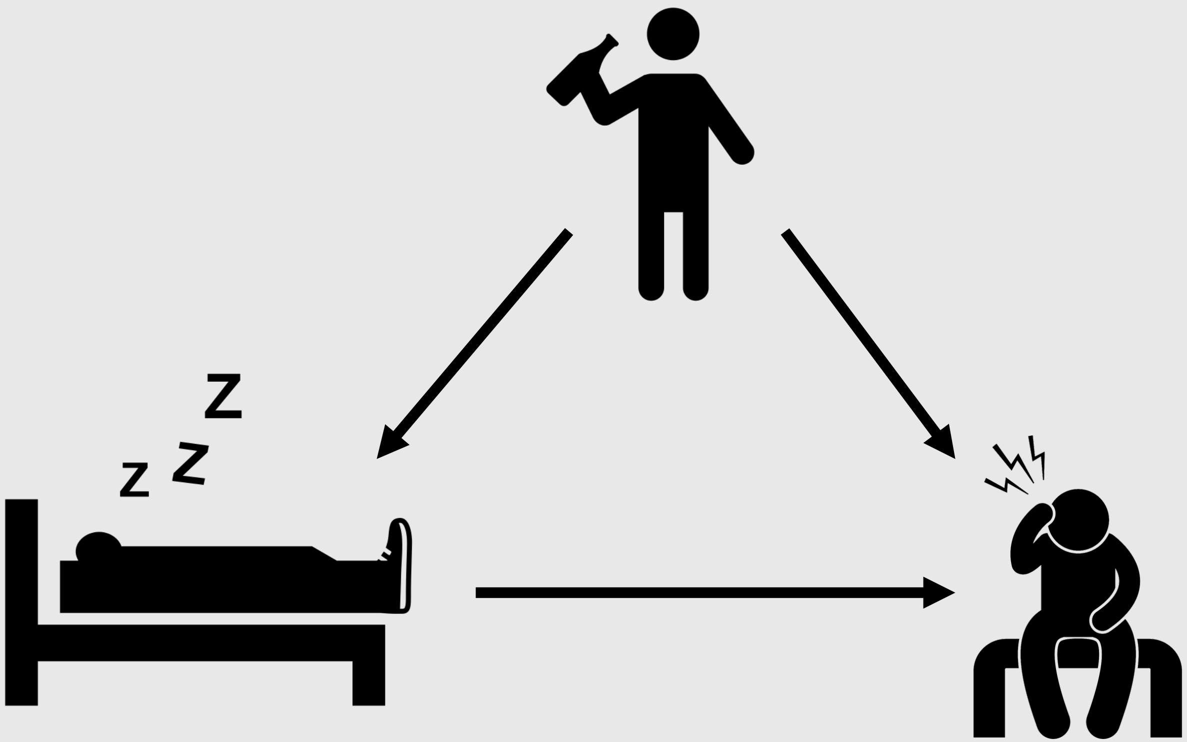

Comparability and covariate balance: intuition

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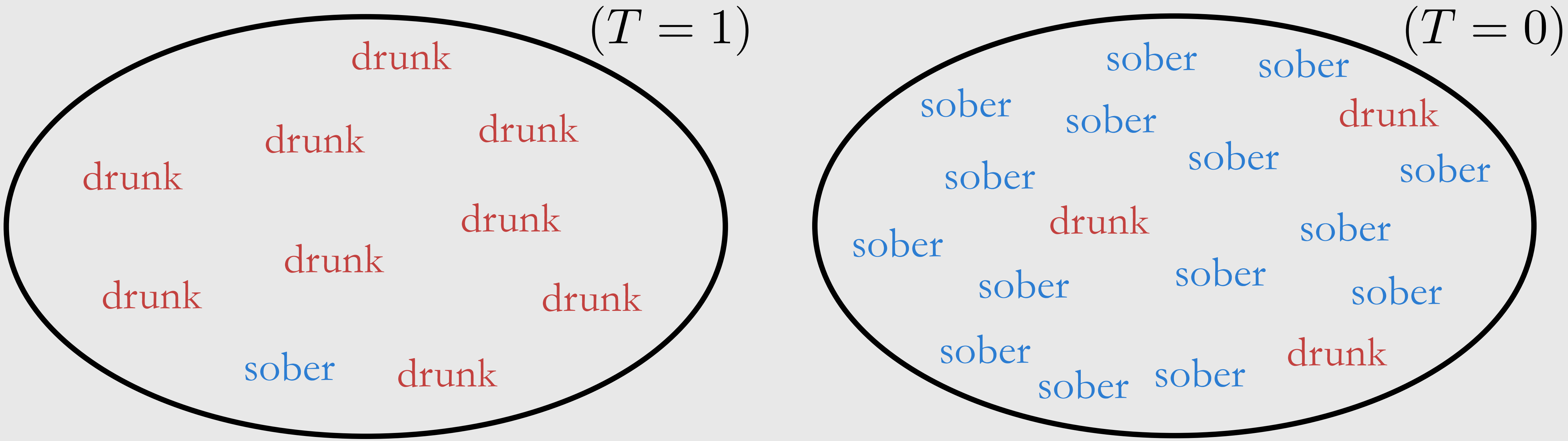

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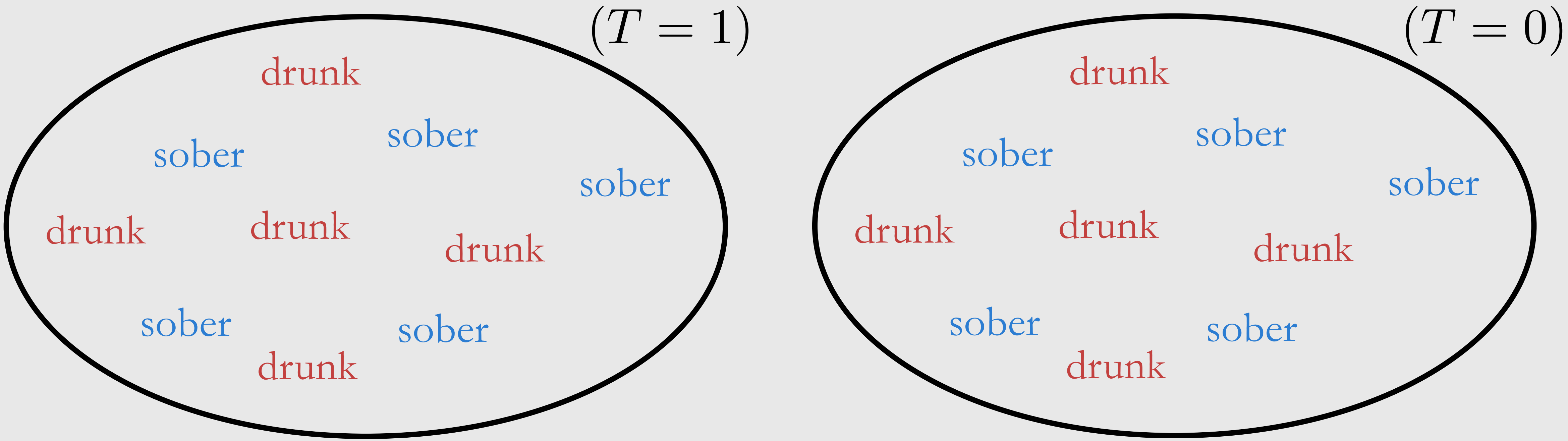

Covariate balance definition

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We have covariate balance if the distribution of covariates X is the same across treatment groups. More formally,

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$$P(X \mid T = 1) \stackrel{d}{=} P(X \mid T = 0)$$

Randomization implies covariate balance

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Because T is not at all determined by X (solely by a coin flip), $T \perp\!\!\!\perp X$

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$$= P(y \mid t)$$

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$P(X \mid T = 1) \overset{d}{=} P(X \mid T = 0)$

Let X be a sufficient adjustment set

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Covariate balance implies association is causation

$P(X \mid T=1) \stackrel{d}{=} P(X \mid T=0)$

Let X be a sufficient adjustment set

$$P(y \mid do(t)) = \sum_x P(y \mid t, x) P(x)$$

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$$P(y \mid do(t)) = \sum_x P(y \mid t, x) P(x)$$

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$$\begin{aligned} P(y \mid do(t)) &= \sum_x P(y \mid t, x) P(x) \\ &= \sum_x \frac{P(y \mid t, x) P(t \mid x) P(x)}{P(t \mid x)} \\ &= \sum_x \frac{P(y, t, x)}{P(t \mid x)} \end{aligned}$$

Covariate balance implies association is causation

$P(X \mid T=1) \stackrel{d}{=} P(X \mid T=0) \implies T \perp\!\!\!\perp X$

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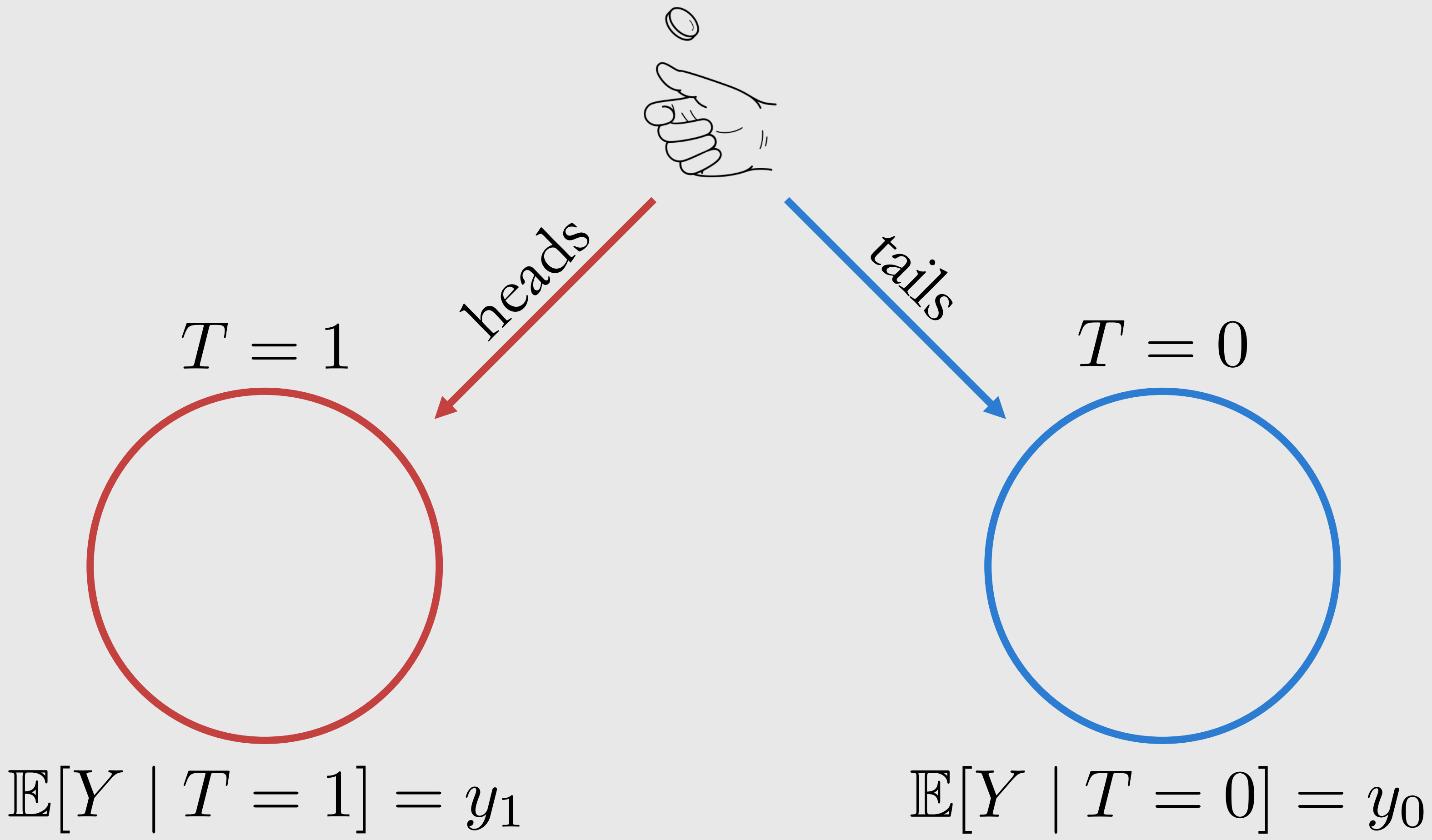
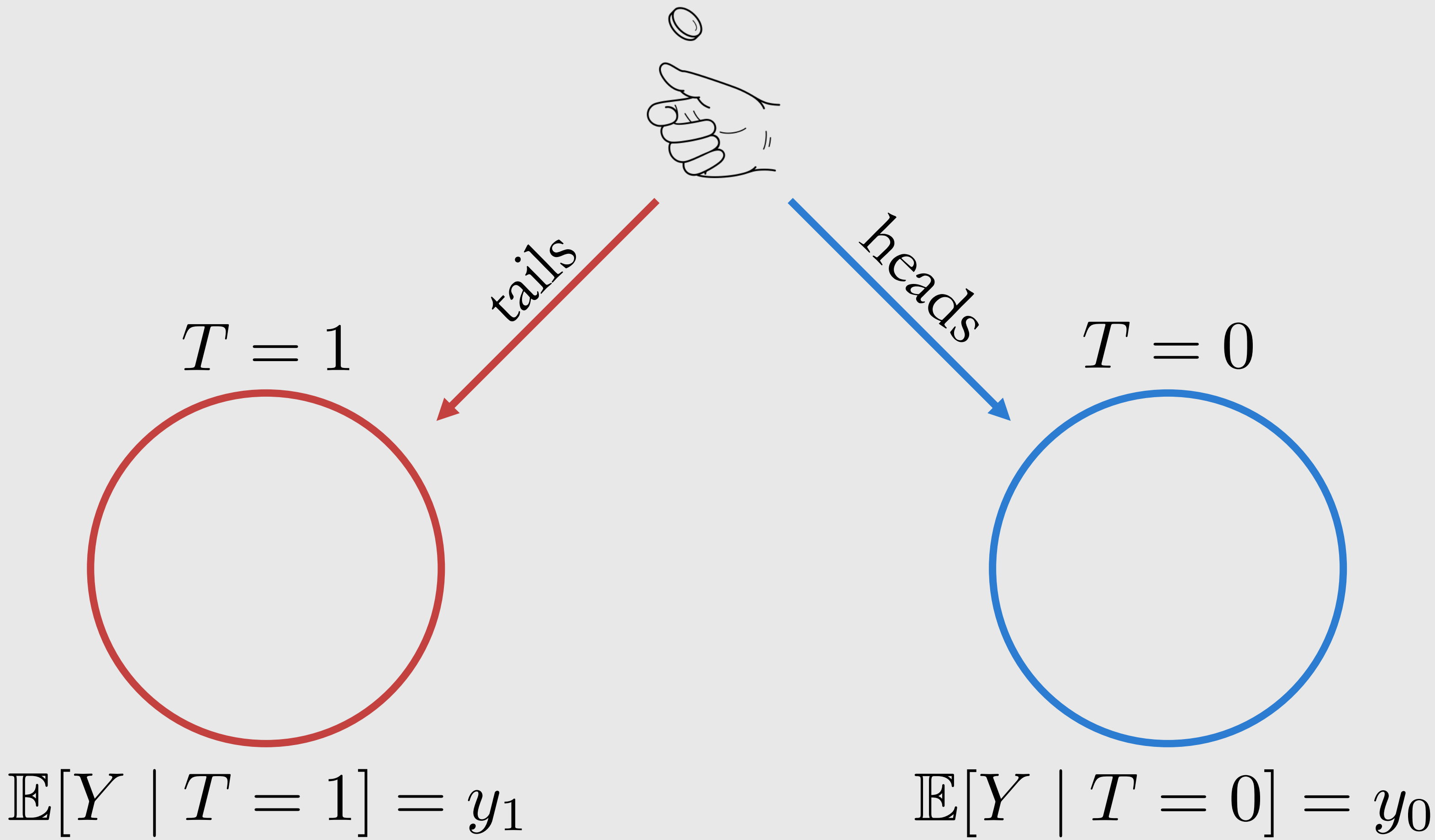
&= P(y \mid t)
\end{aligned}$$

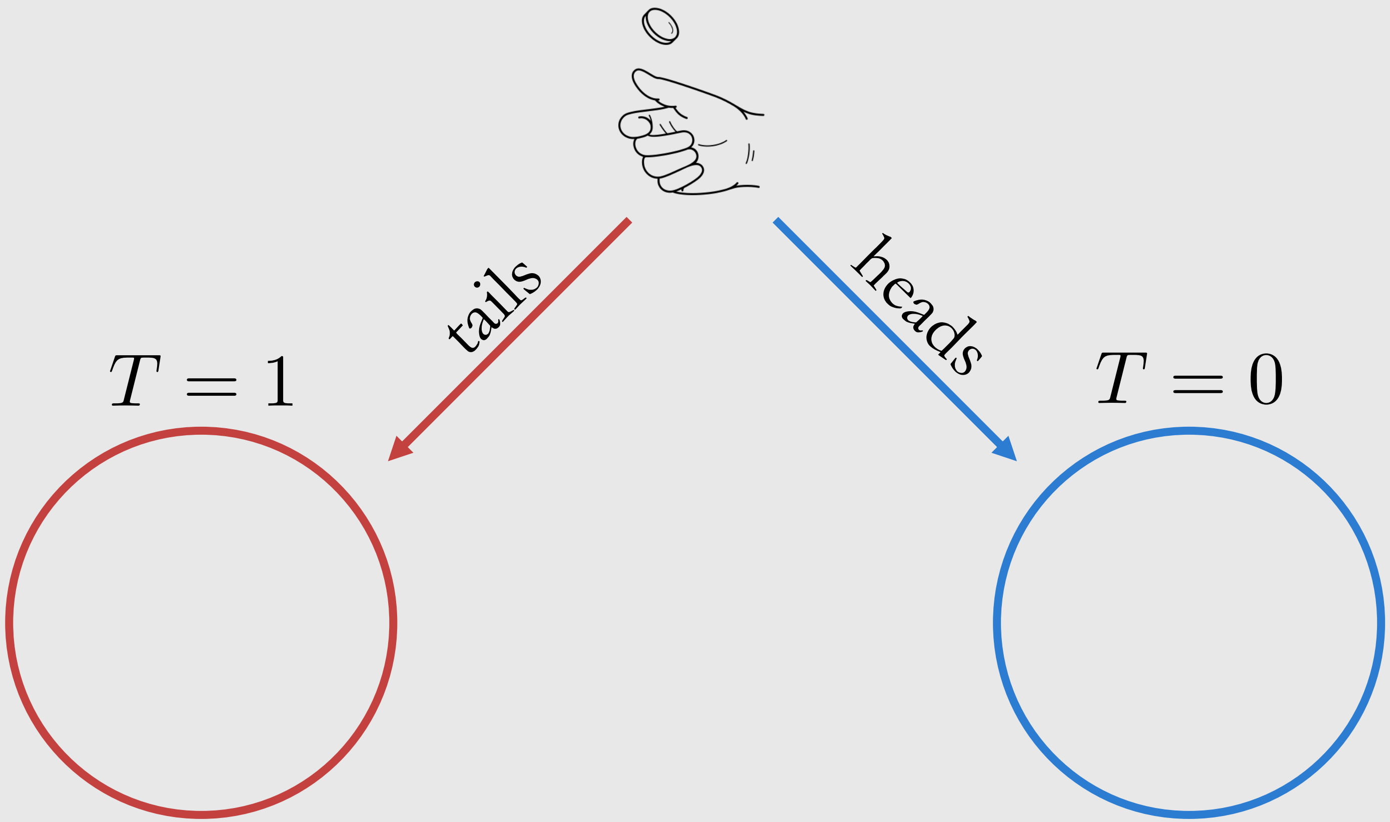
Exchangeability



Exchangeability



Exchangeability



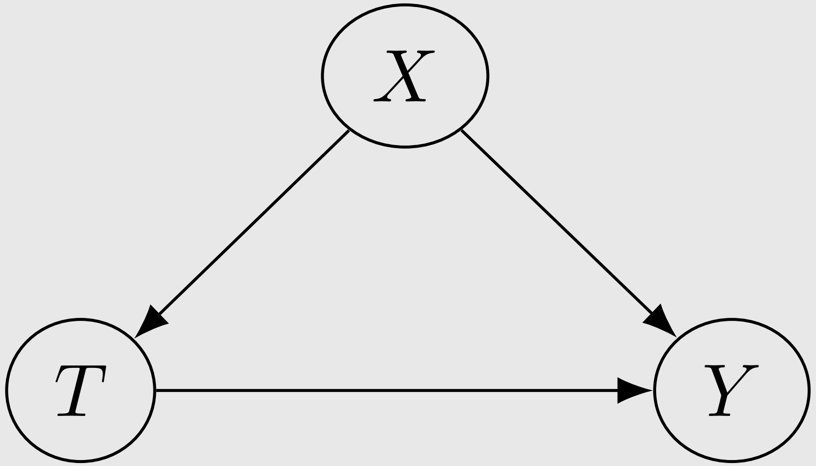
$\mathbb{E}[Y \mid T = 1] = y_1$

$\mathbb{E}[Y \mid T = 0] = y_0$

Question:

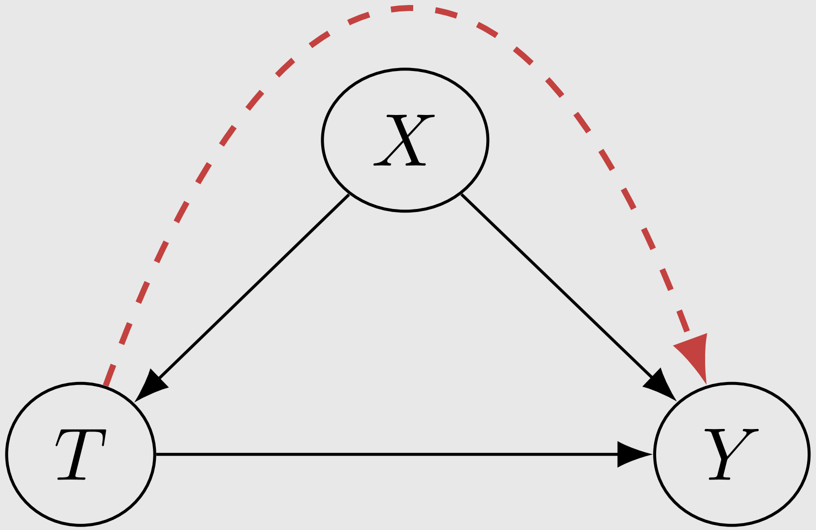
Write down the formal definition of (mean) exchangeability. Then, prove that this yields "association is causation."

No backdoor paths



No backdoor paths

Confounding association



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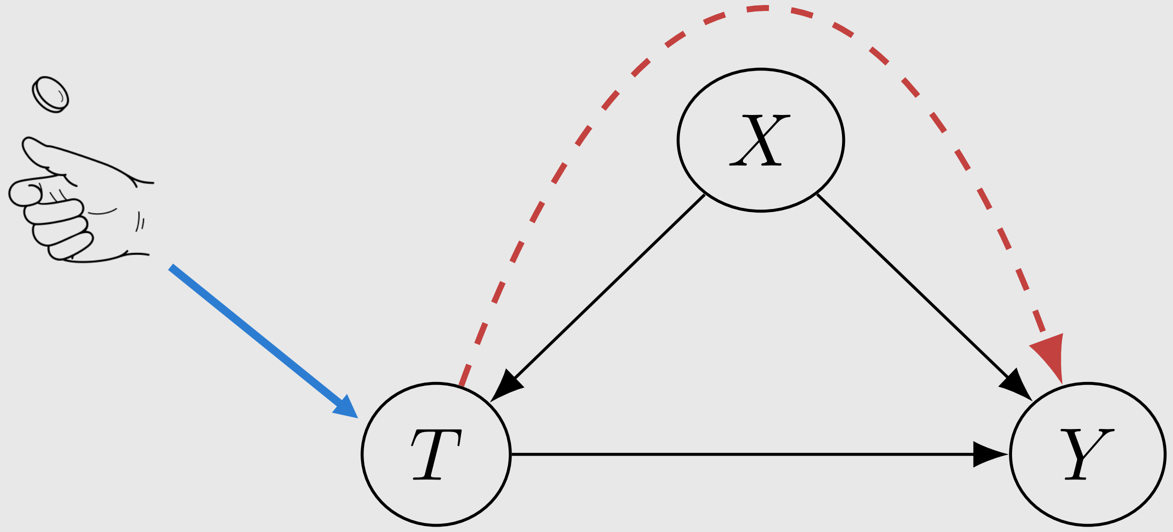

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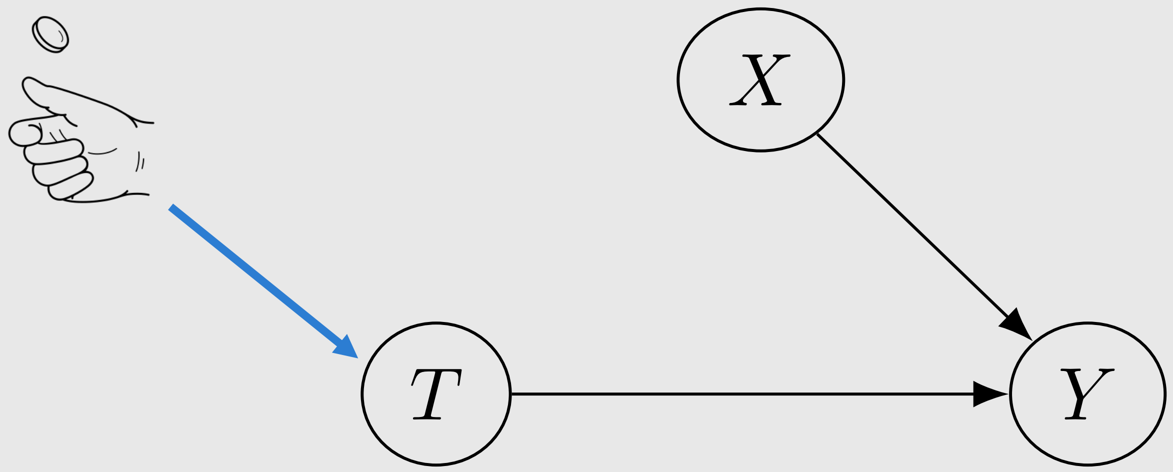
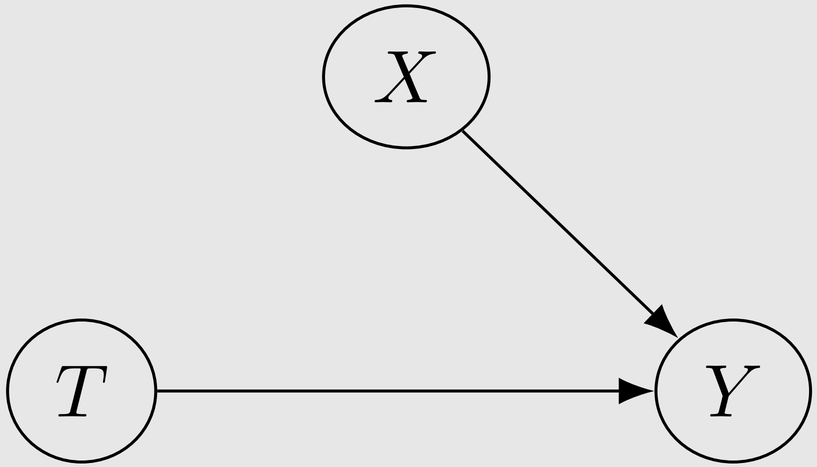


Question:
What previous result tells us that association is causation in this graph?



The magic of randomized experiments

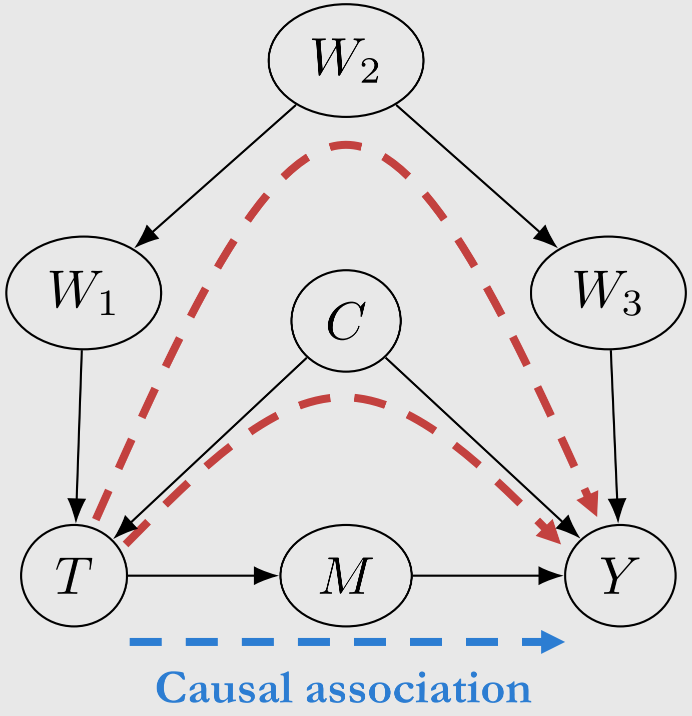
Frontdoor adjustment

Pearl's *do*-calculus

Determining identifiability from the graph

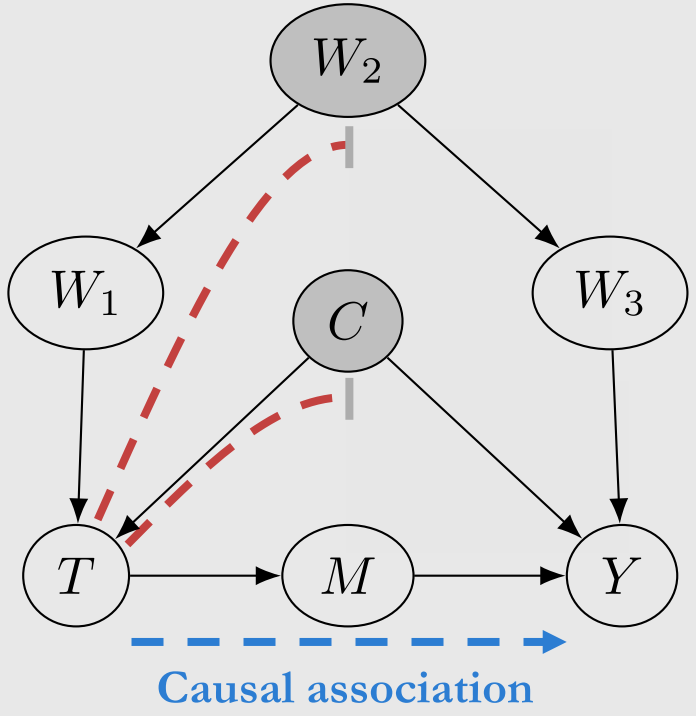
Recall the backdoor adjustment

$P(Y \mid t)$



Recall the backdoor adjustment

$$P(Y \mid t, c, w_2)$$



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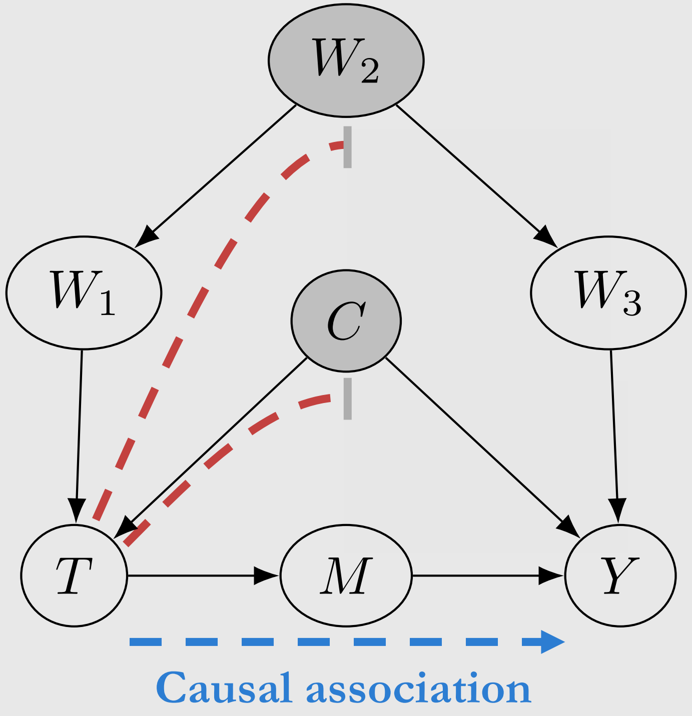
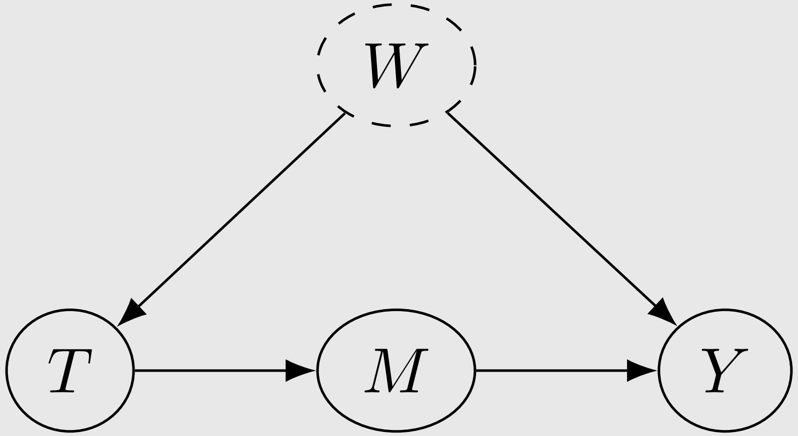
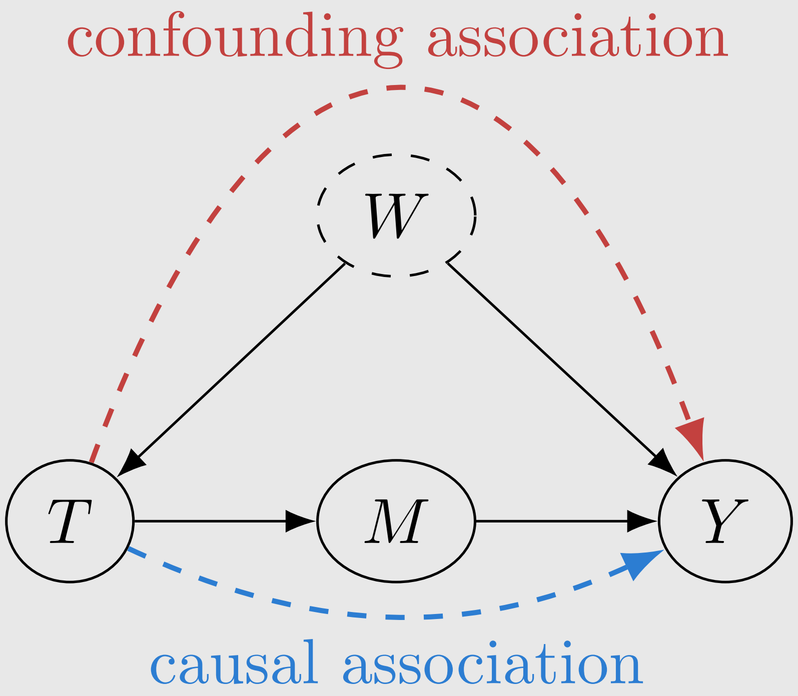




Frontdoor adjustment: big picture



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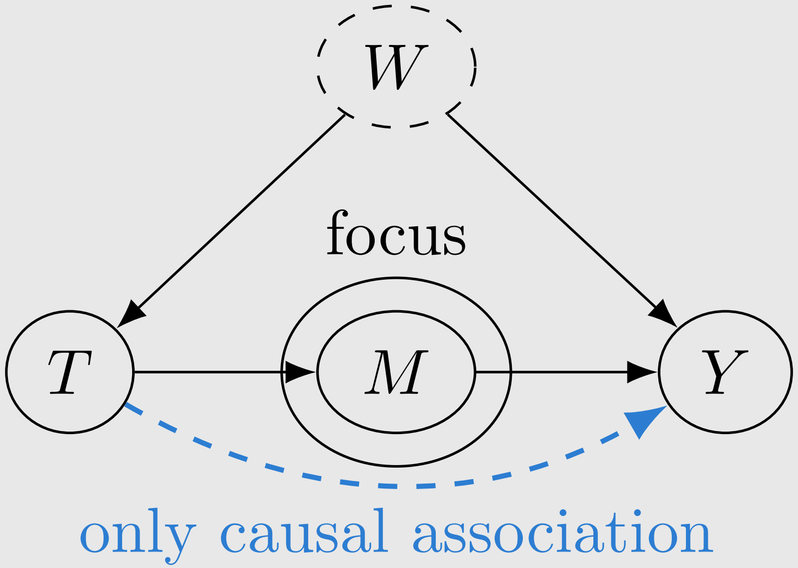

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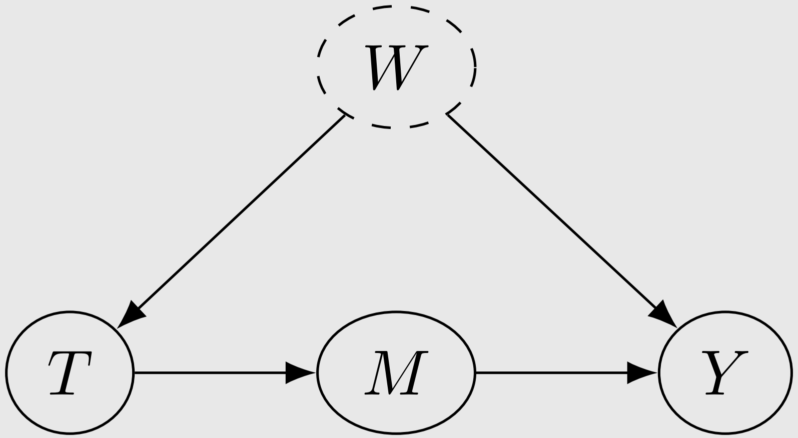
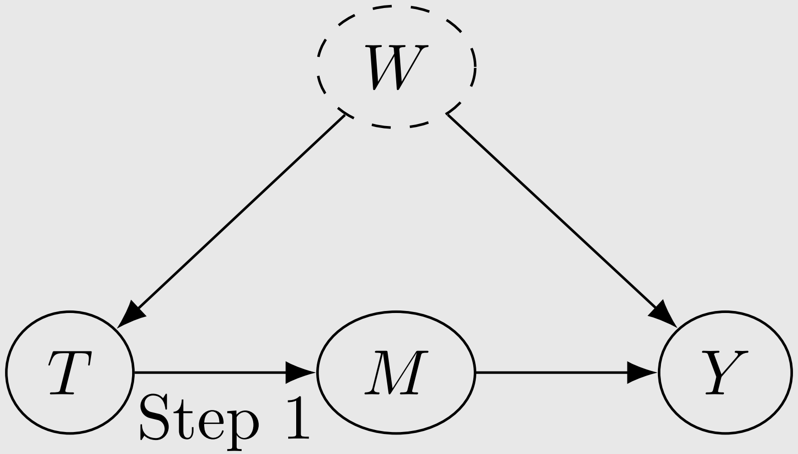


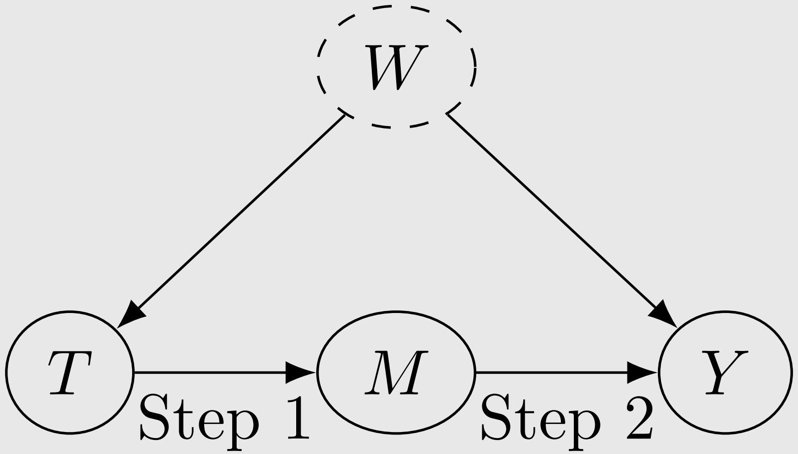
Frontdoor adjustment: big picture

1. Identify the causal effect of T on M



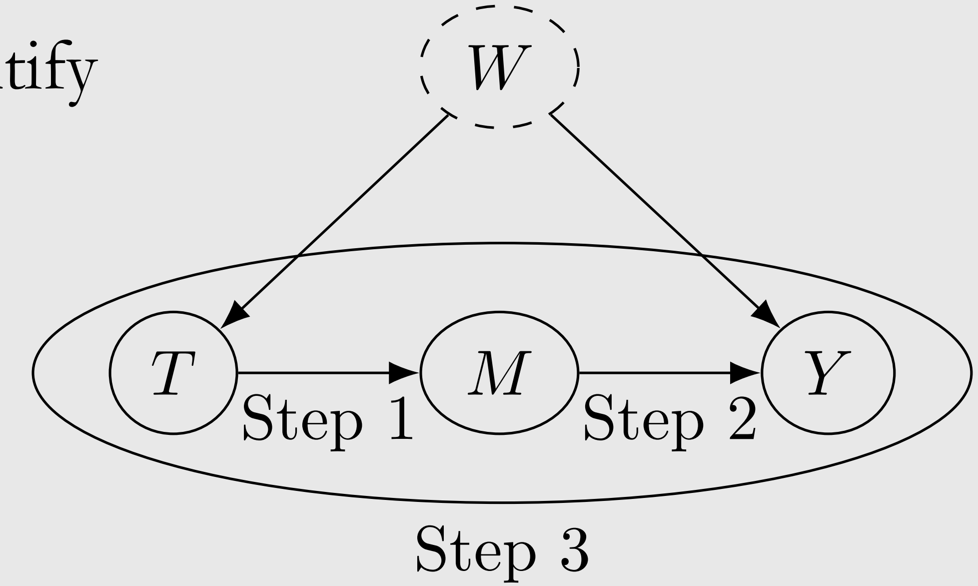
Frontdoor adjustment: big picture

1. Identify the causal effect of T on M
2. Identify the causal effect of M on Y



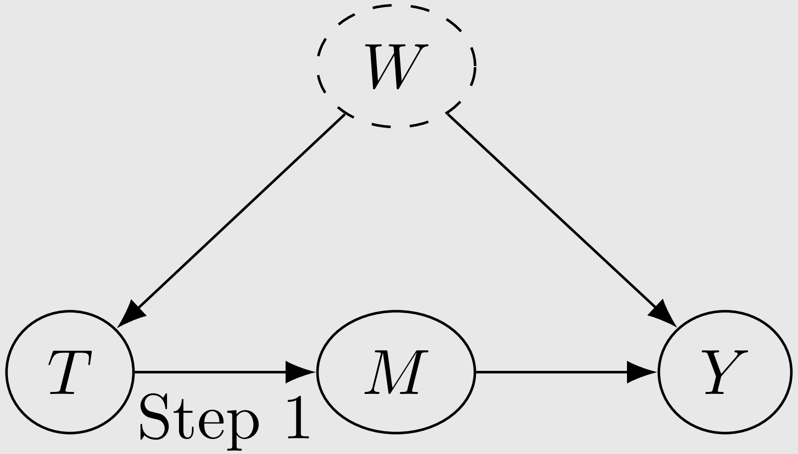
Frontdoor adjustment: big picture

1. Identify the causal effect of T on M
2. Identify the causal effect of M on Y
3. Combine the above steps to identify the causal effect of T on Y



Frontdoor adjustment: step 1

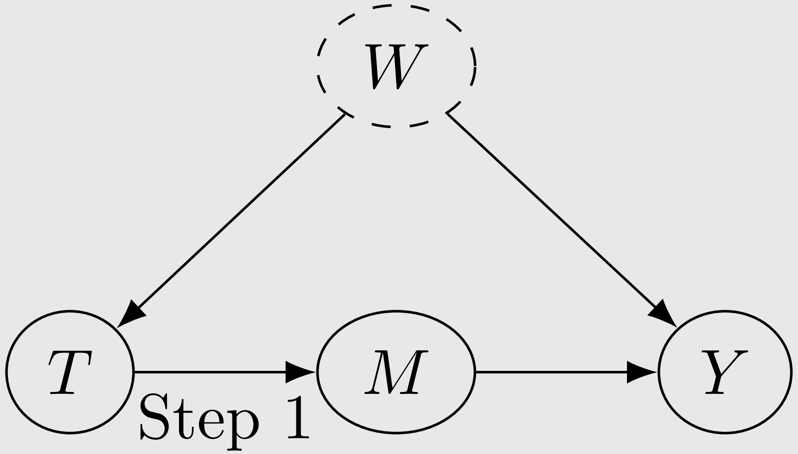
Identify the causal effect of T on M



Frontdoor adjustment: step 1

Identify the causal effect of T on M

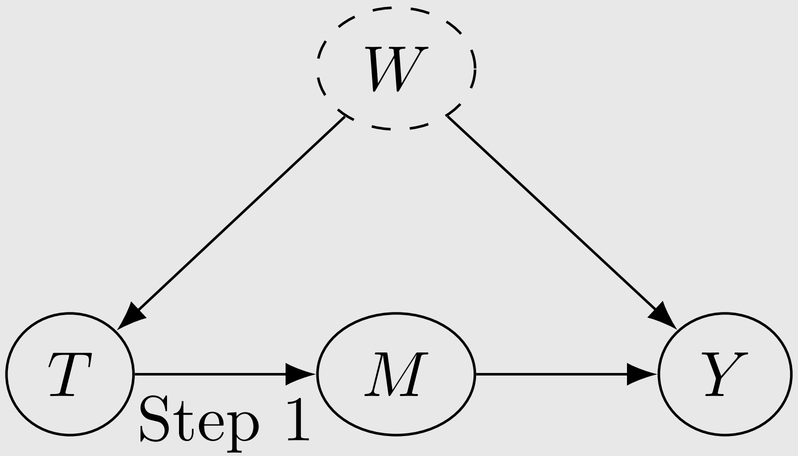
$$P(m \mid do(t))$$



Frontdoor adjustment: step 1

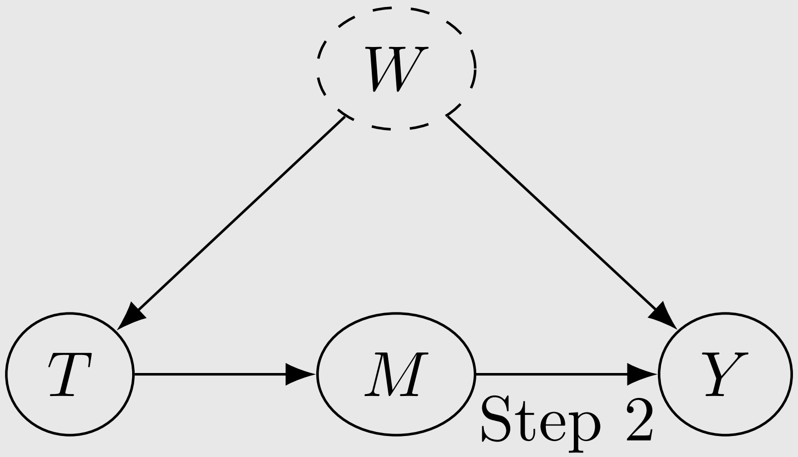
Identify the causal effect of T on M

$$P(m \mid do(t)) = P(m \mid t)$$



Frontdoor adjustment: step 2

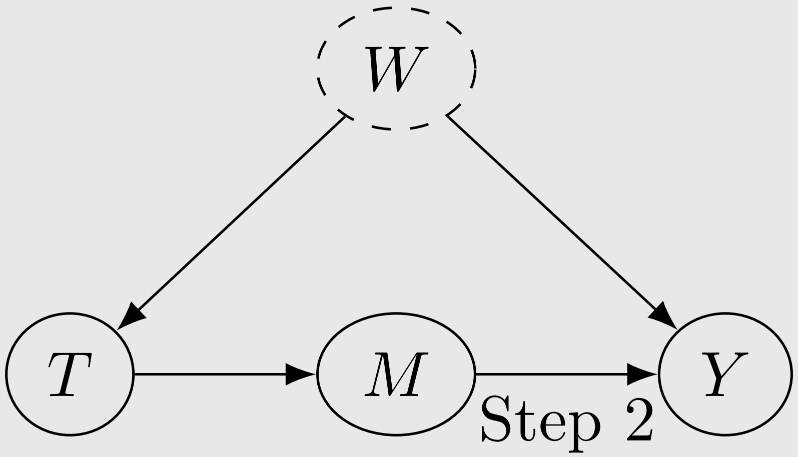
Identify the causal effect of M on Y



Frontdoor adjustment: step 2

Identify the causal effect of M on Y

$$P(y \mid do(m))$$



Frontdoor adjustment: step 2

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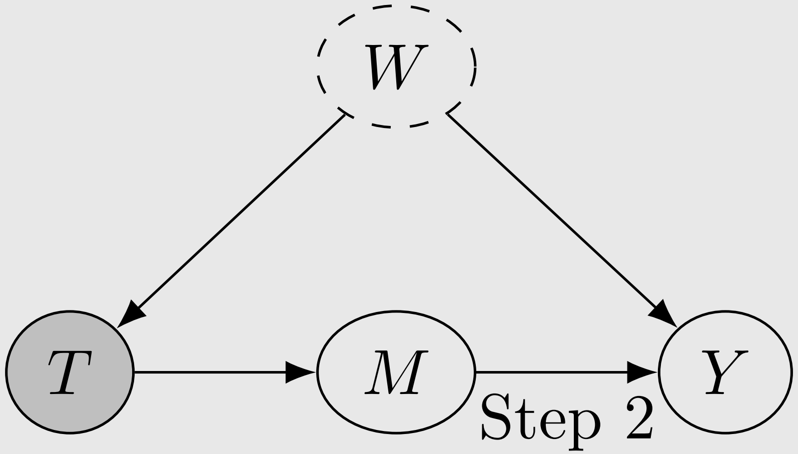
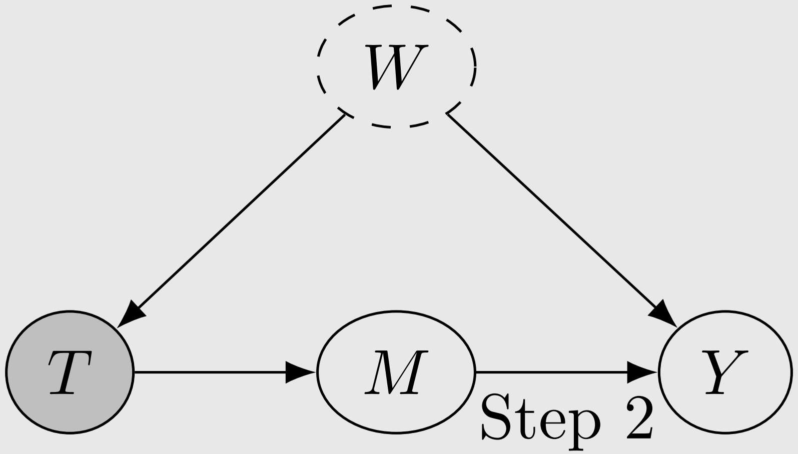


Frontdoor adjustment: step 2

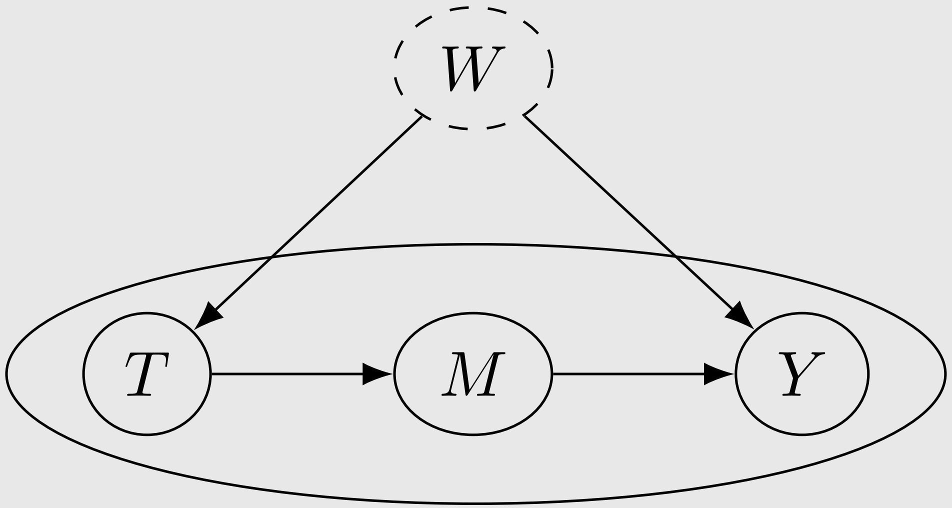
Identify the causal effect of M on Y

$$P(y \mid do(m)) = \sum_t P(y \mid m, t)\, P(t)$$



Frontdoor adjustment: step 3

Combine steps 1 and 2 to identify the causal effect of T on Y



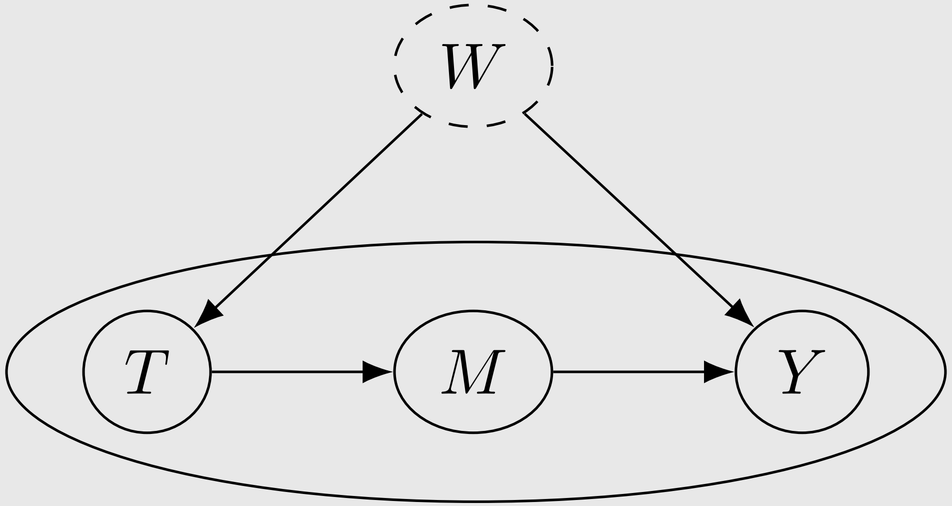
Step 3

Frontdoor adjustment: step 3

Combine steps 1 and 2 to identify the causal effect of T on Y

Goal

$P(y \mid do(t))$



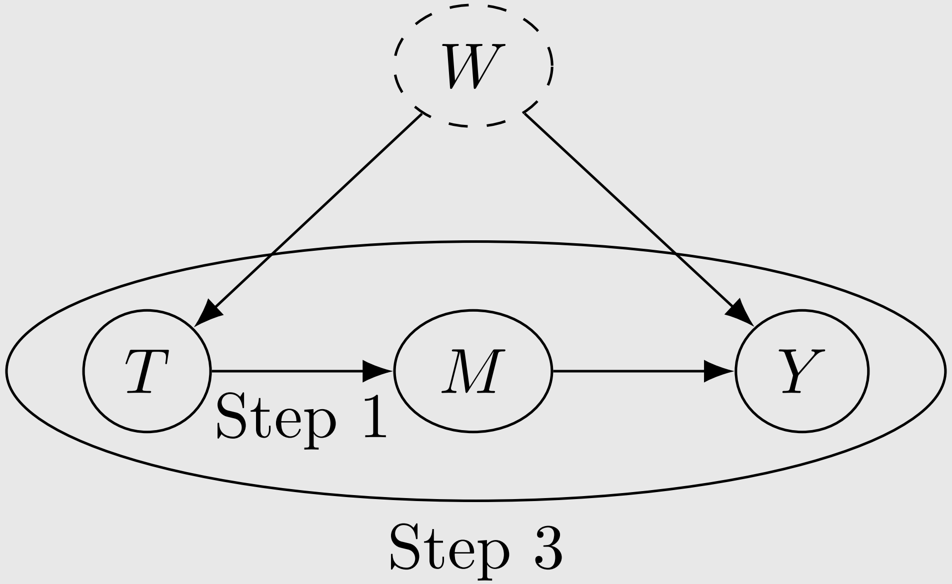
Step 3

Frontdoor adjustment: step 3

Combine steps 1 and 2 to identify the causal effect of T on Y

Goal

$P(y \mid do(t))$ $\quad\quad$ $P(m \mid do(t))$

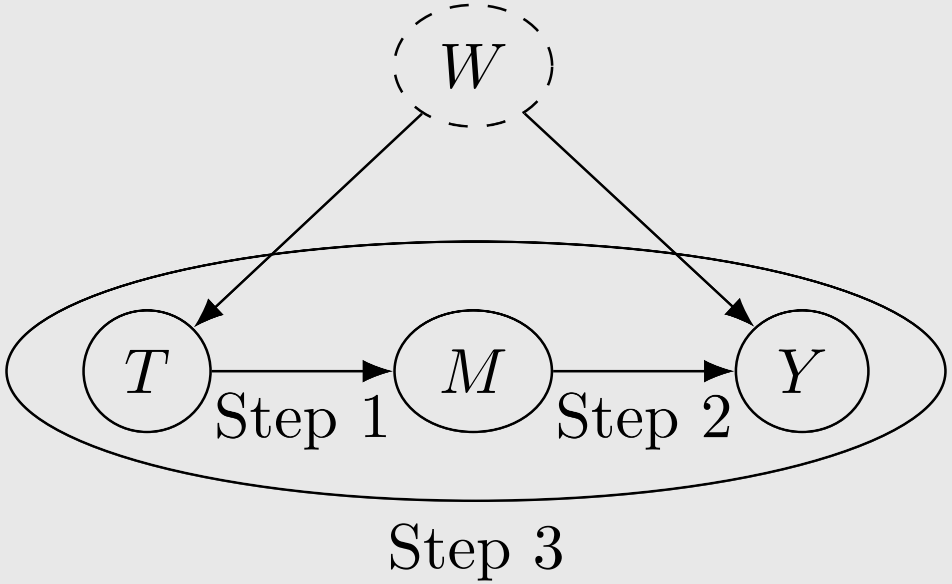


Frontdoor adjustment: step 3

Combine steps 1 and 2 to identify the causal effect of T on Y

Goal

$P(y \mid do(t))$ $\qquad$ $P(m \mid do(t))\, P(y \mid do(m))$



Frontdoor adjustment: step 3

Combine steps 1 and 2 to identify the causal effect of T on Y

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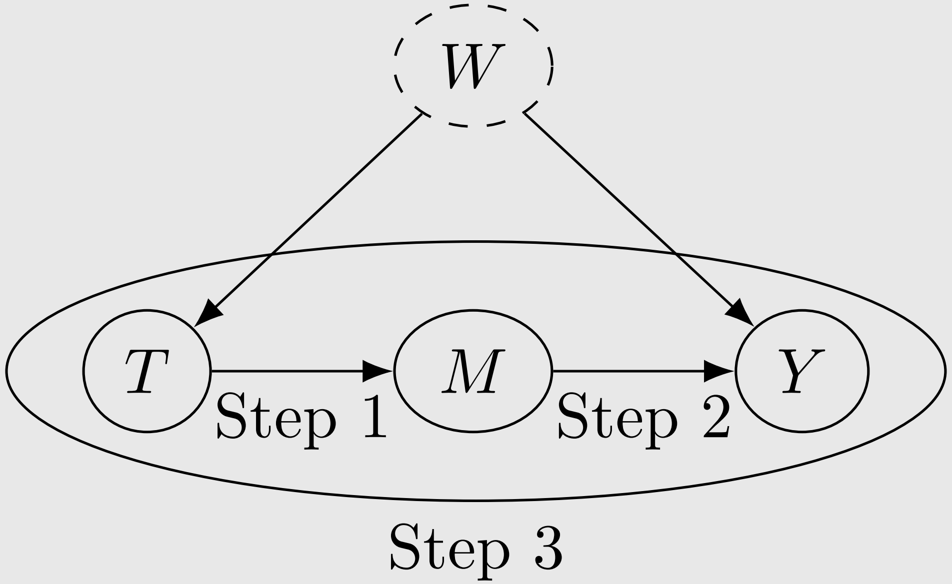
$$P(y \mid do(t)) = \sum_m P(m \mid do(t))\, P(y \mid do(m))$$



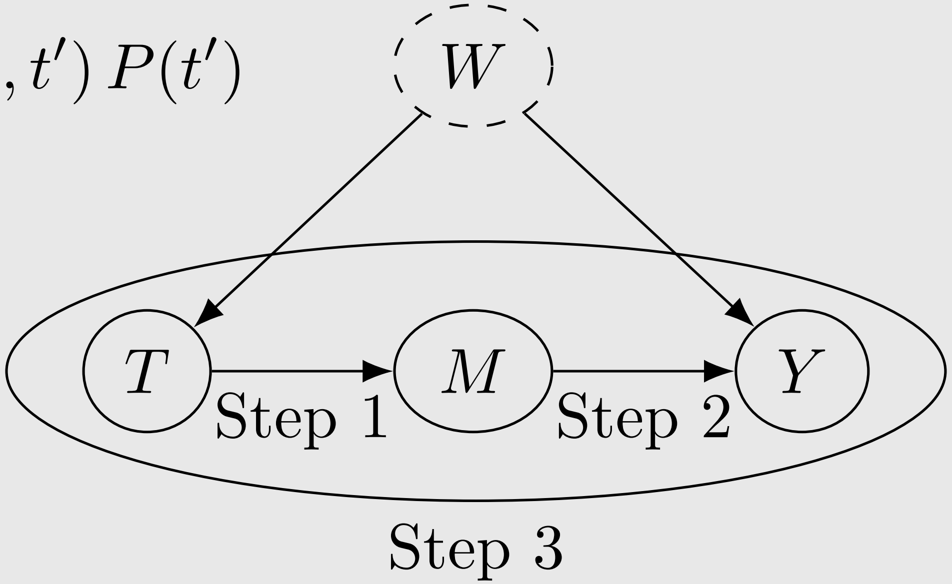
Frontdoor adjustment: step 3

Combine steps 1 and 2 to identify the causal effect of T on Y

Goal

$$P(y \mid do(t)) = \sum_m P(m \mid do(t))\, P(y \mid do(m))$$

$$= \sum_m P(m \mid t) \sum_{t'} P(y \mid m, t')\, P(t')$$



The frontdoor adjustment and criterion

$$P(y \mid do(t)) = \sum_m P(m \mid t) \sum_{t'} P(y \mid m, t') \, P(t')$$

The frontdoor adjustment and criterion

If (T, M, Y) satisfy the frontdoor criterion, and we have positivity, then

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A set of variables M satisfies the **frontdoor criterion** relative to T and Y if the following are true:

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A set of variables M satisfies the **frontdoor criterion** relative to T and Y if the following are true:

1. M completely mediates the effect of T on Y (i.e. all causal paths from T to Y go through M).

The frontdoor adjustment and criterion

If (T, M, Y) satisfy the frontdoor criterion, and we have positivity, then

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A set of variables M satisfies the **frontdoor criterion** relative to T and Y if the following are true:

1. M completely mediates the effect of T on Y (i.e. all causal paths from T to Y go through M).
2. There is no unblocked backdoor path from T to M.

The frontdoor adjustment and criterion

If (T, M, Y) satisfy the frontdoor criterion, and we have positivity, then

$$P(y \mid do(t)) = \sum_m P(m \mid t) \sum_{t'} P(y \mid m, t') \, P(t')$$

A set of variables M satisfies the **frontdoor criterion** relative to T and Y if the following are true:

1. M completely mediates the effect of T on Y (i.e. all causal paths from T to Y go through M).
2. There is no unblocked backdoor path from T to M.
3. All backdoor paths from M to Y are blocked by T.

See proof of frontdoor adjustment using the truncated factorization in Section 6.1 of the course book

Question:
What is the intuition for why the frontdoor criterion gives us identifiability?

The magic of randomized experiments

Frontdoor adjustment

Pearl's *do*-calculus

Determining identifiability from the graph

Can we identify the causal effect if neither the backdoor criterion nor the frontdoor criterion is satisfied?

Yes, and *do*-calculus tells us how.

Pearl's *do*-calculus

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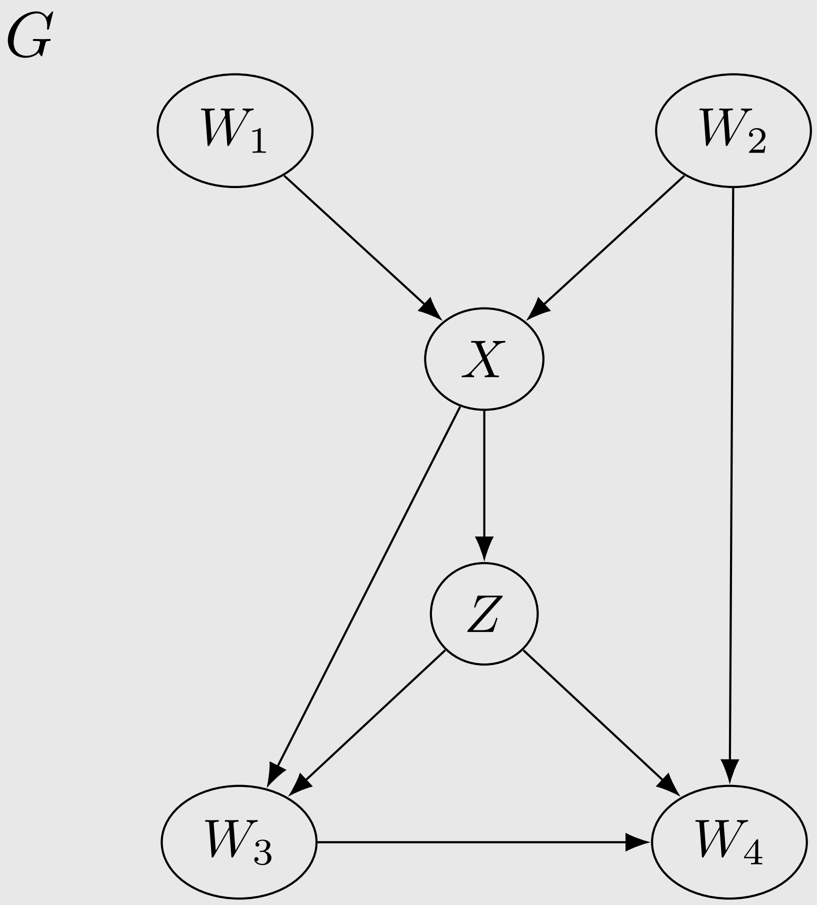
Multiple treatments and/or multiple outcomes

Notation for Pearl's *do*-calculus



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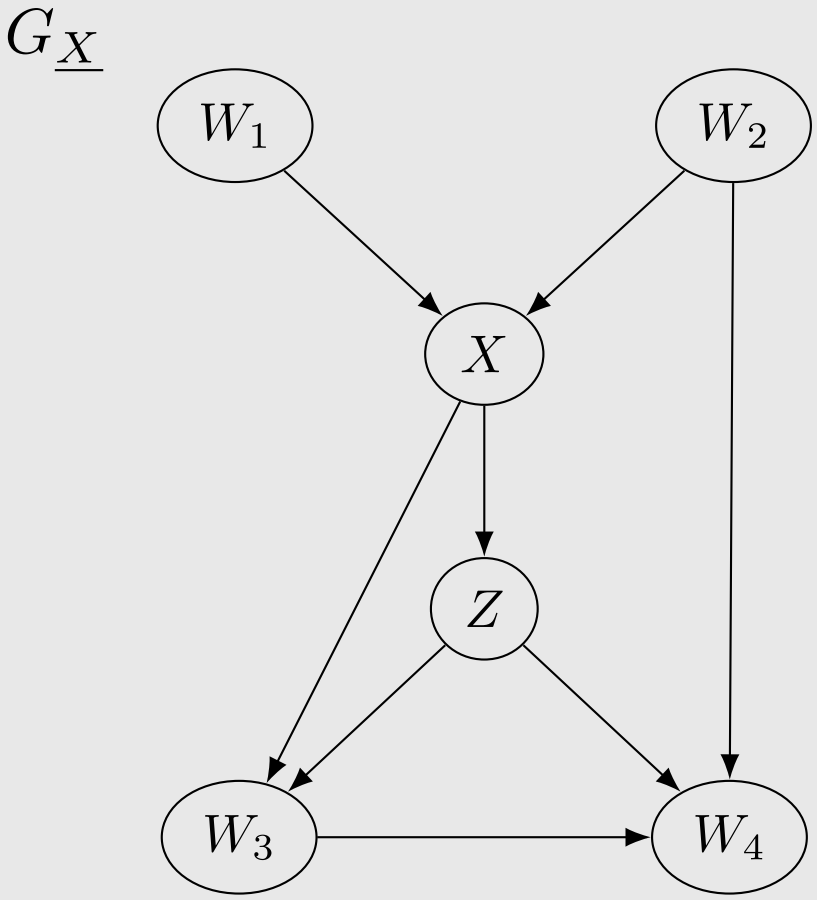

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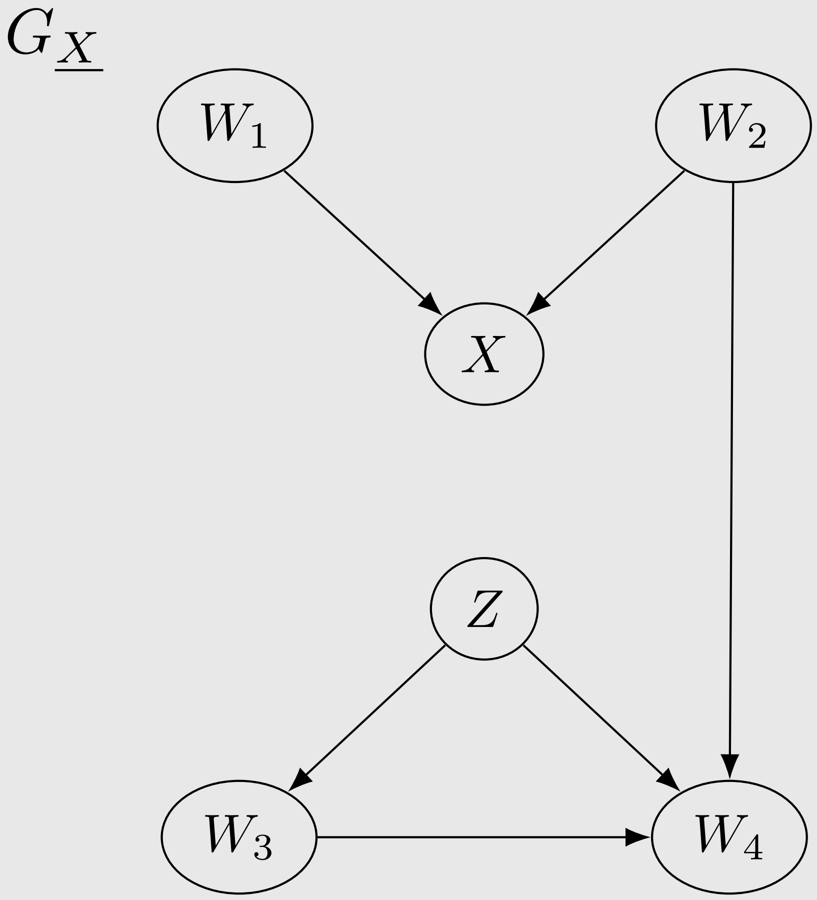

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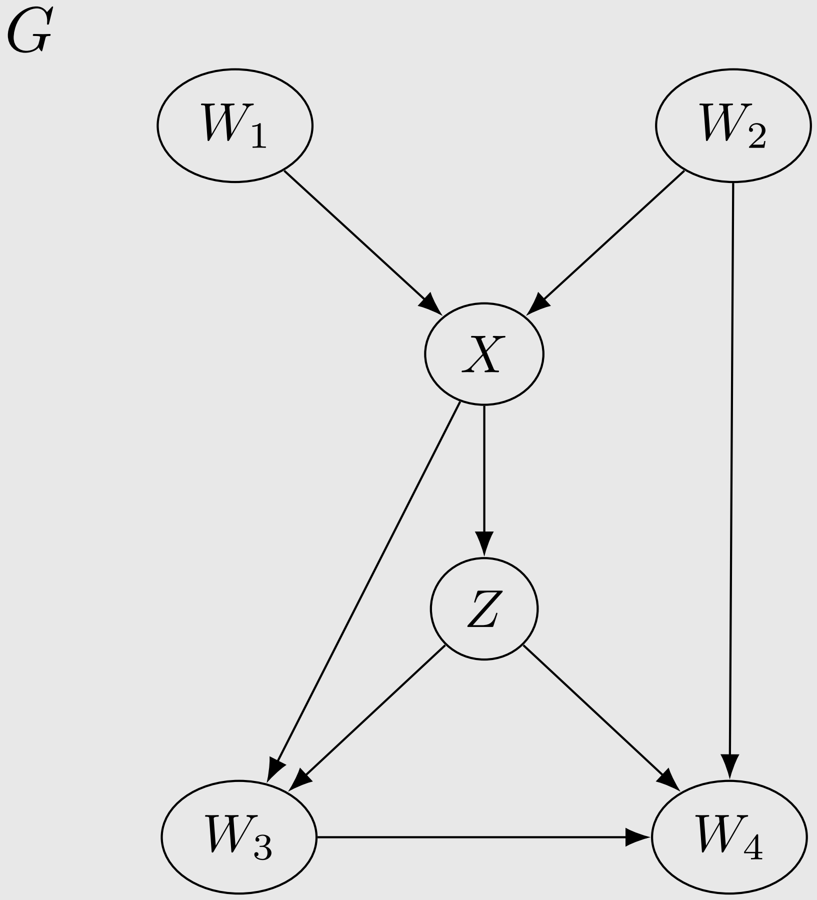

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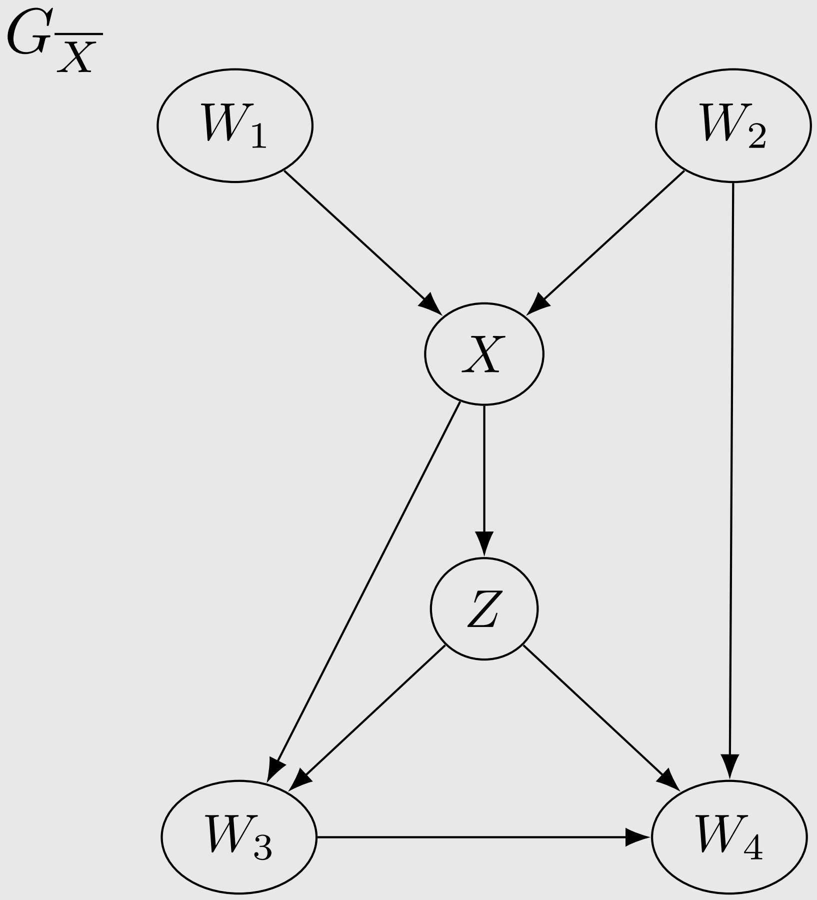

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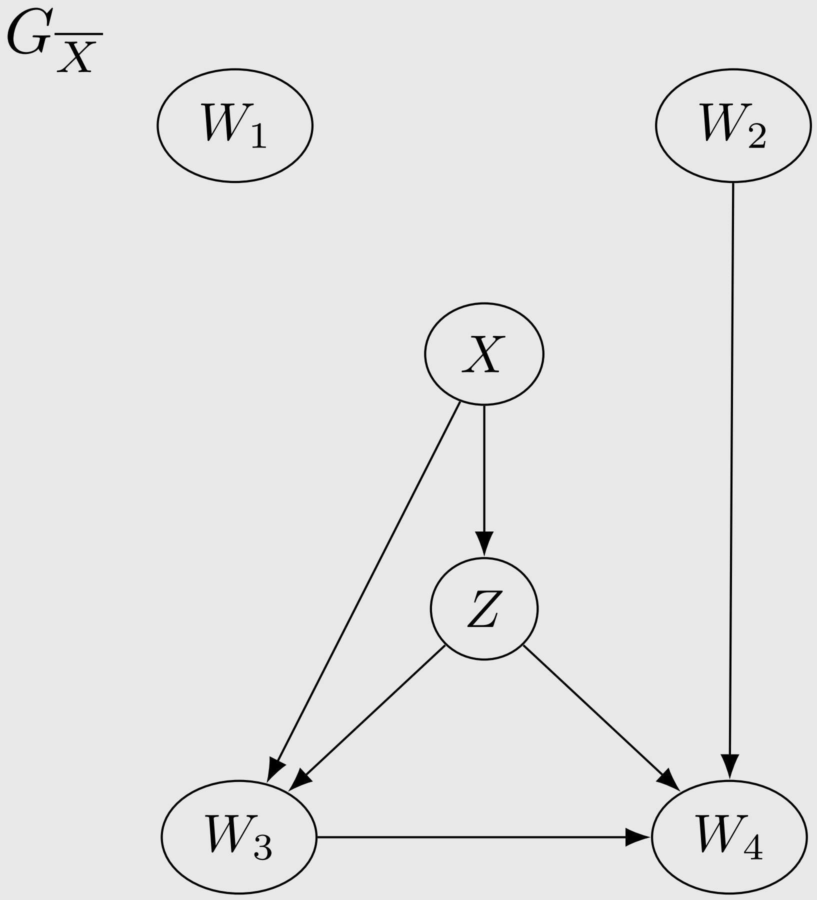
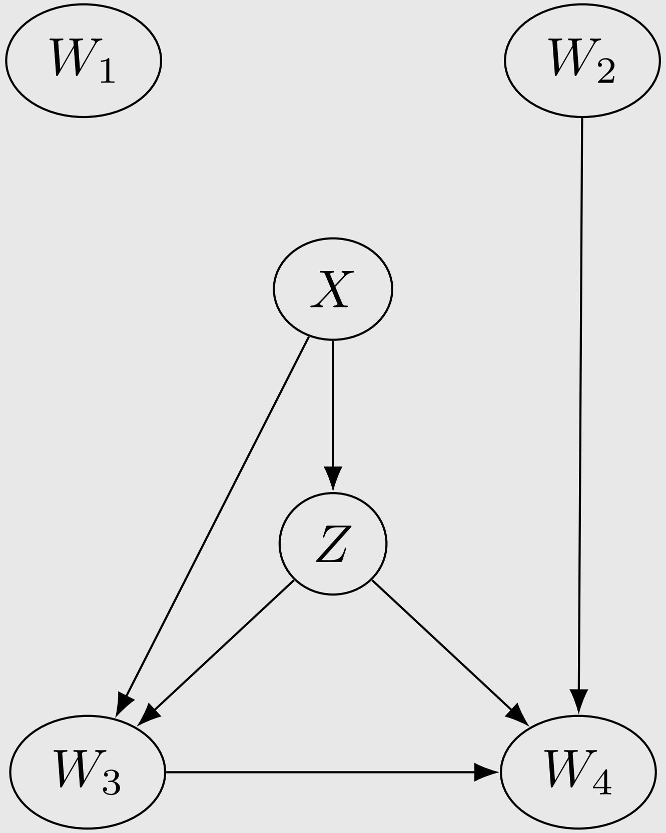


Notation for Pearl's *do*-calculus

$G_{\overline{X}\underline{Z}}$



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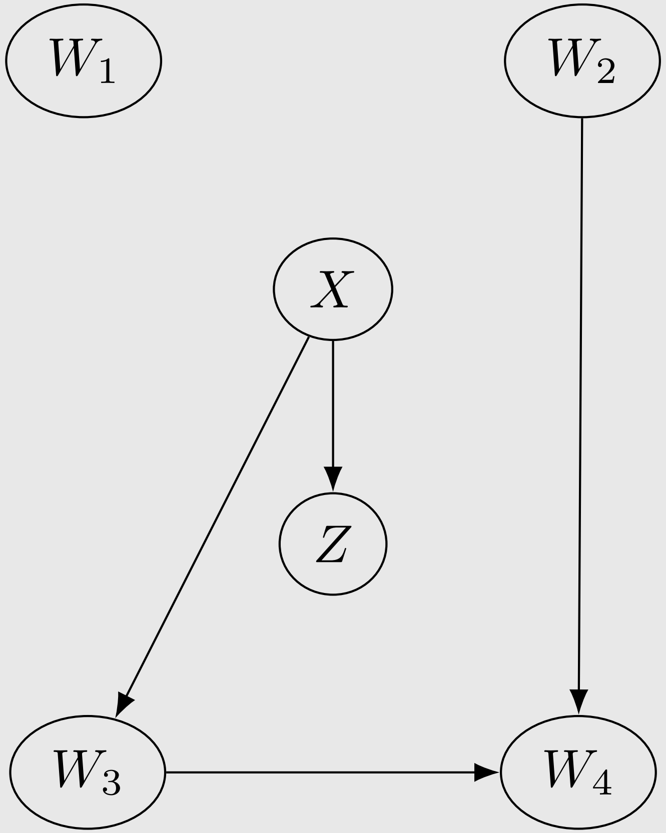

Rule 1 of *do*-calculus

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$$P(y \mid do(t), z, w) = P(y \mid do(t), w) \quad \text{if } Y \perp\!\!\!\perp_{G_{\overline{T}}} Z \mid T, W$$

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Generalization of d-separation to interventional distributions

Rule 2 of *do*-calculus

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Generalization of backdoor adjustment/criterion

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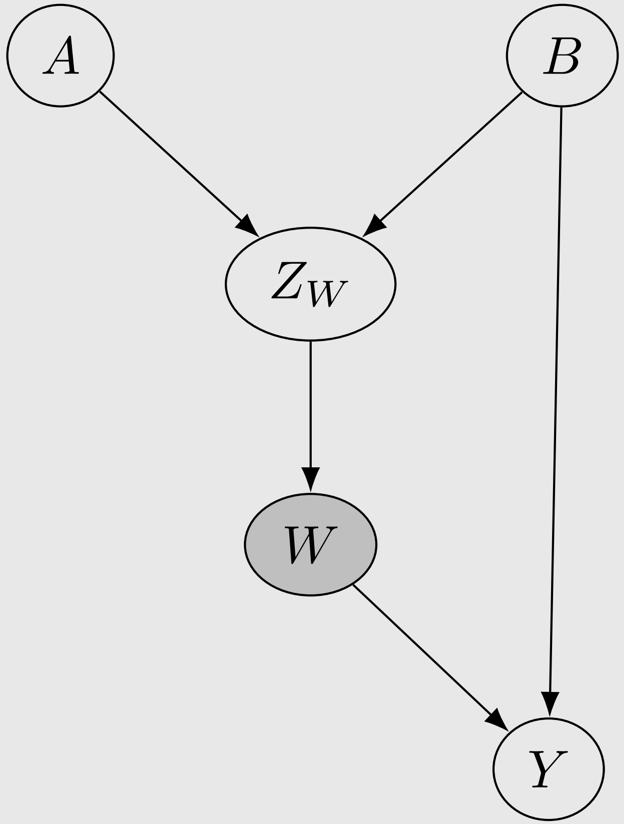

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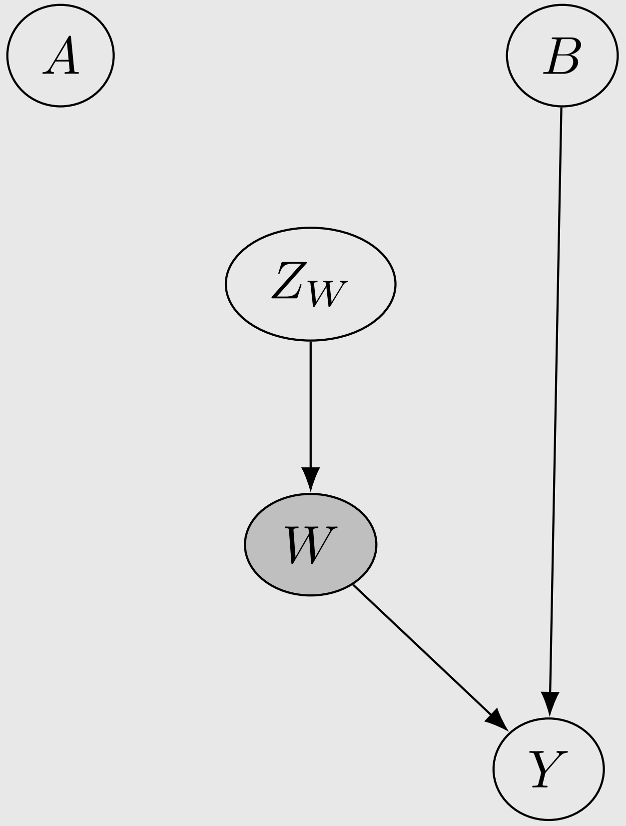

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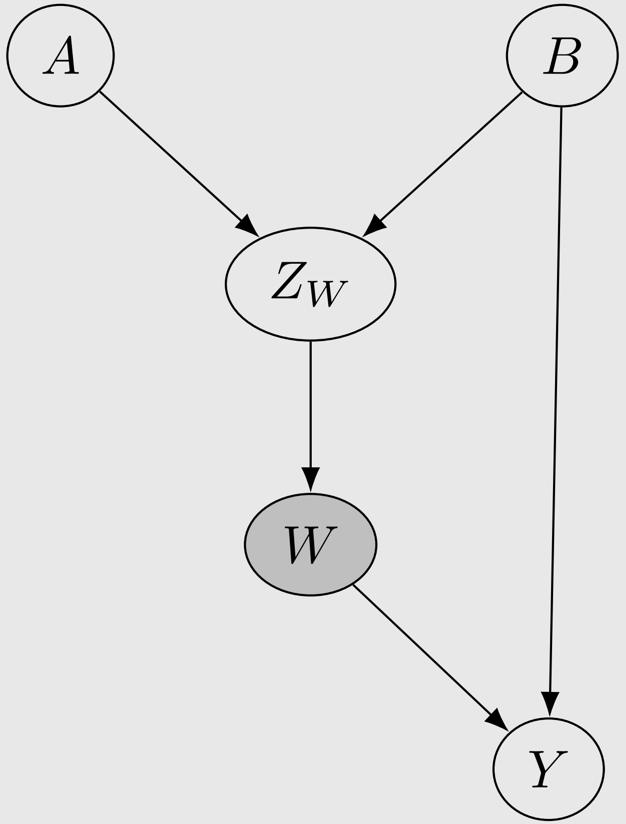

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The rules of *do*-calculus

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Proof of the frontdoor adjustment using *do*-calculus in Section 6.2.1 of the course book (compare with proof using truncated factorization in Section 6.1)

Completeness of *do*-calculus

Maybe there are some identifiable causal estimands that can't be identified using the rules of *do*-calculus

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Constructive proofs that admit polynomial time algorithms for identification

Question:
What concepts are the first and second rules of *do*-calculus generalizations of?

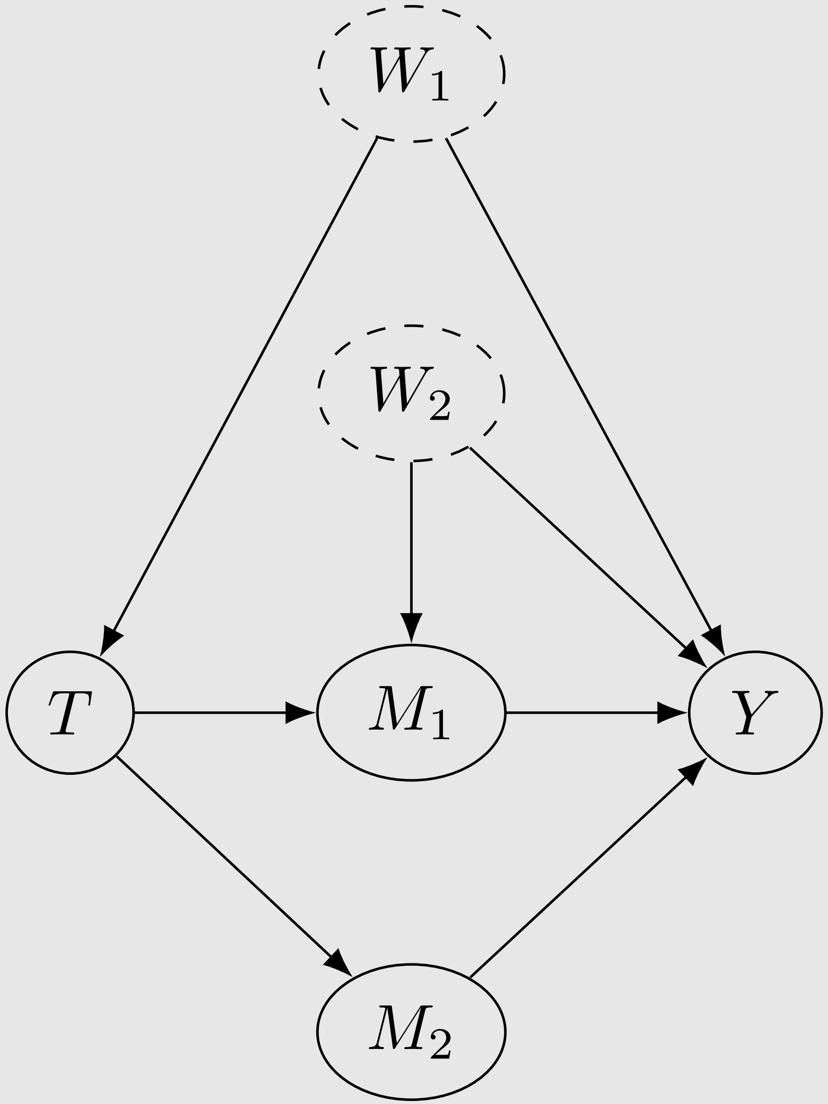
The magic of randomized experiments

Frontdoor adjustment

Pearl's *do*-calculus

Determining identifiability from the graph

Question:
In this graph, is the backdoor criterion satisfied?



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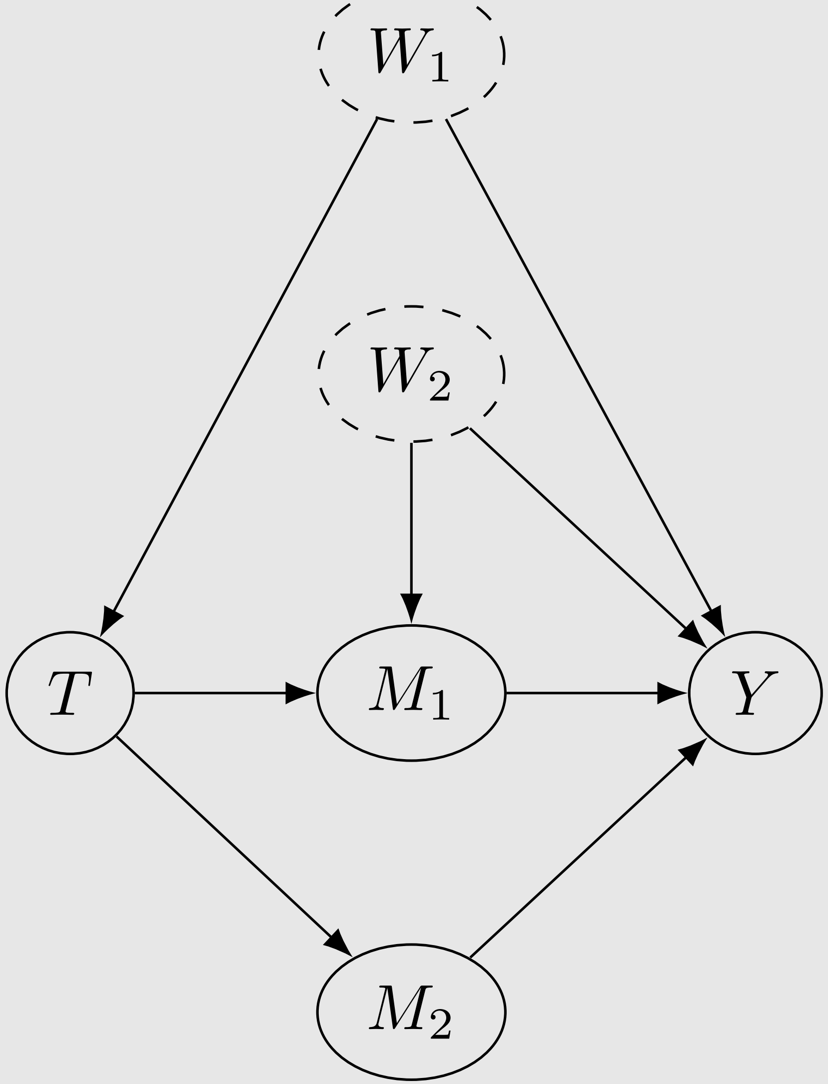
How about the frontdoor criterion?



Unconfounded children criterion

This criterion is satisfied if it is possible to block all backdoor paths from the treatment variable T to all of its children that are ancestors of Y with a single conditioning set (Tian & Pearl, 2002).

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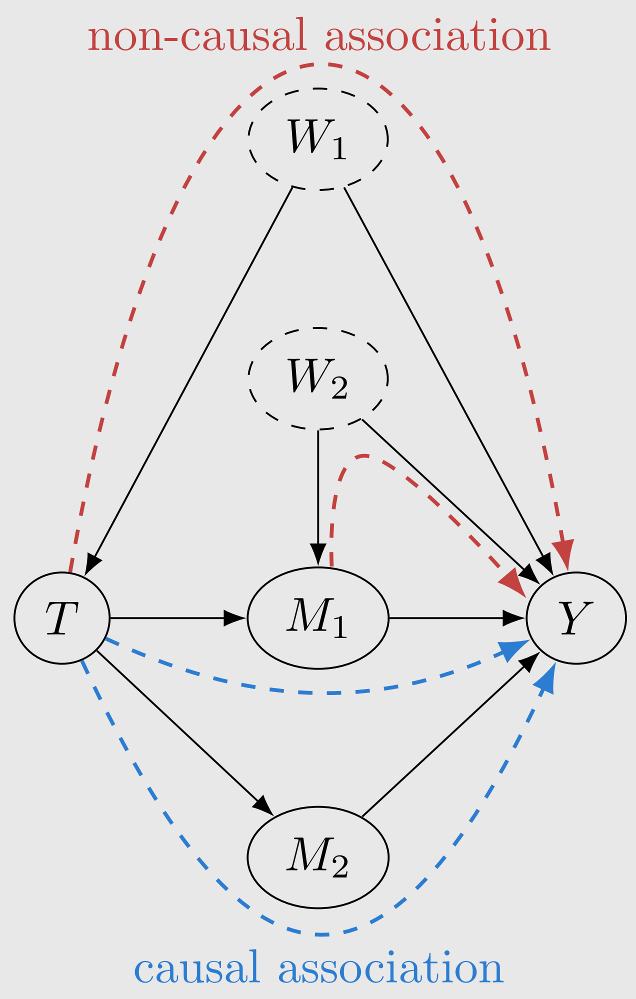

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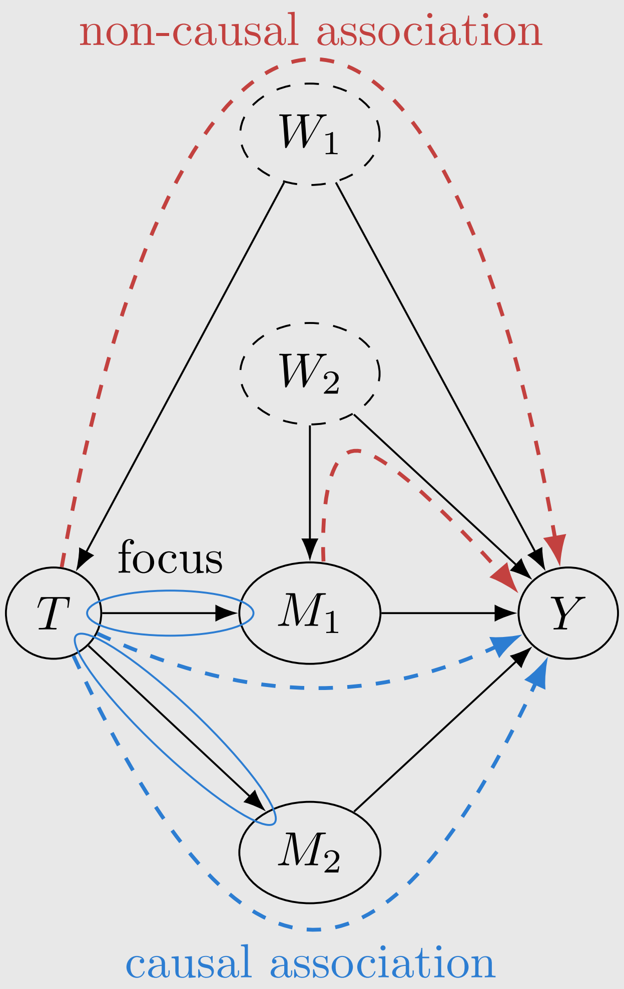

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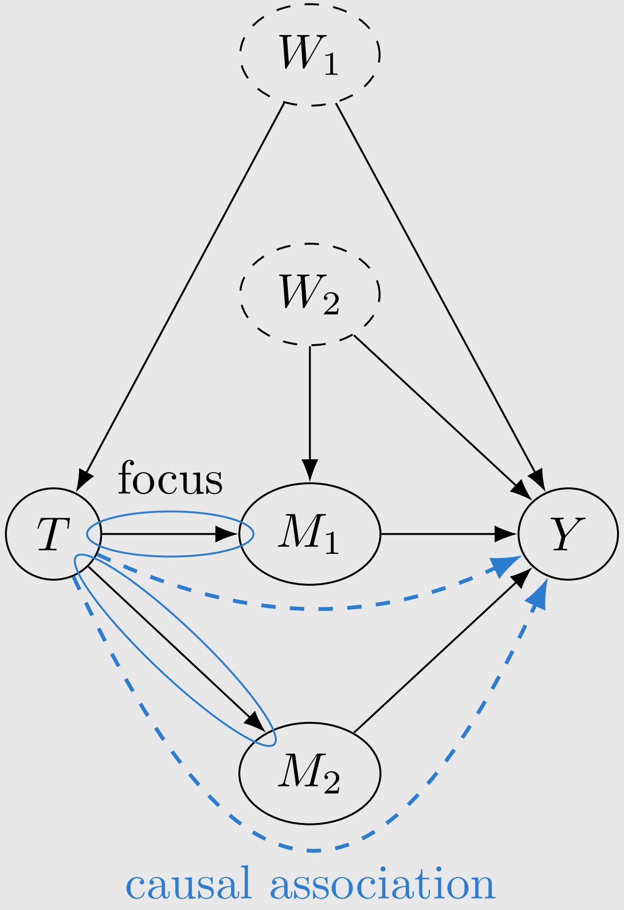

Necessary condition for identifiability

For each backdoor path from T to any child M of T that is an ancestor of Y, it is possible to block that path (Pearl, 2009, p. 92).

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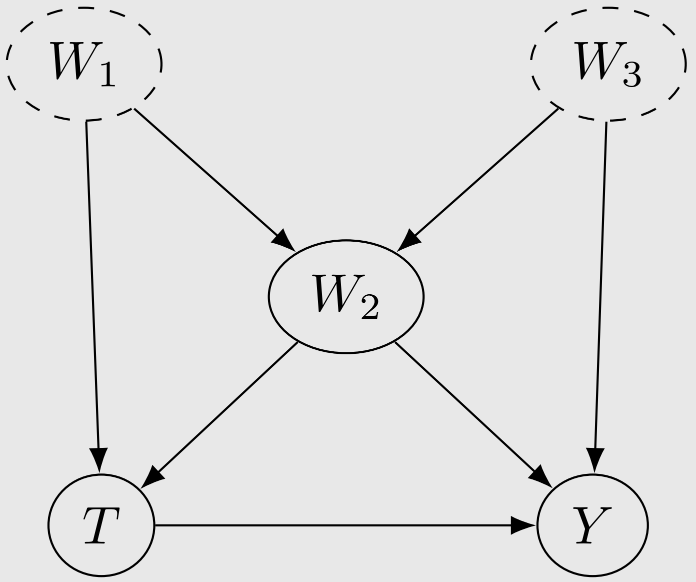

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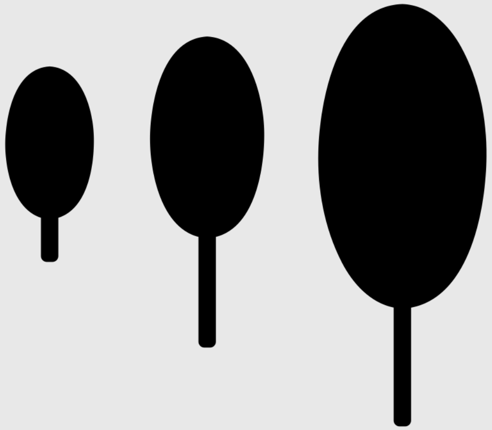

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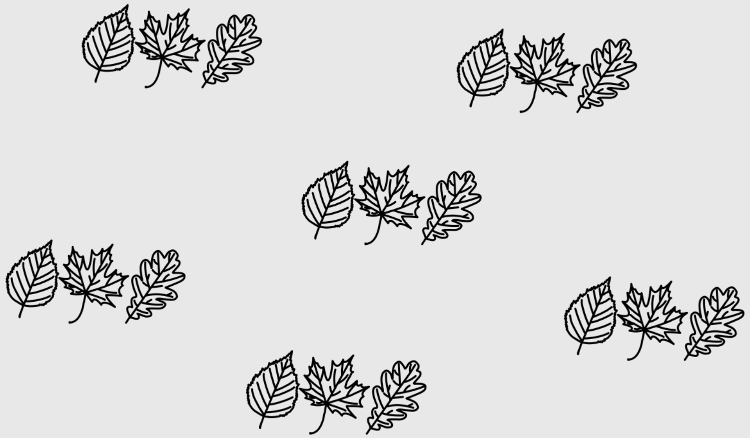
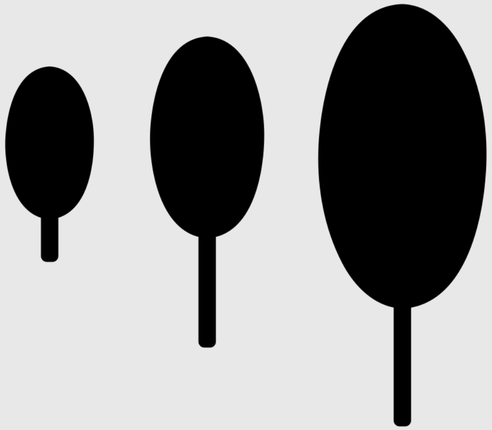
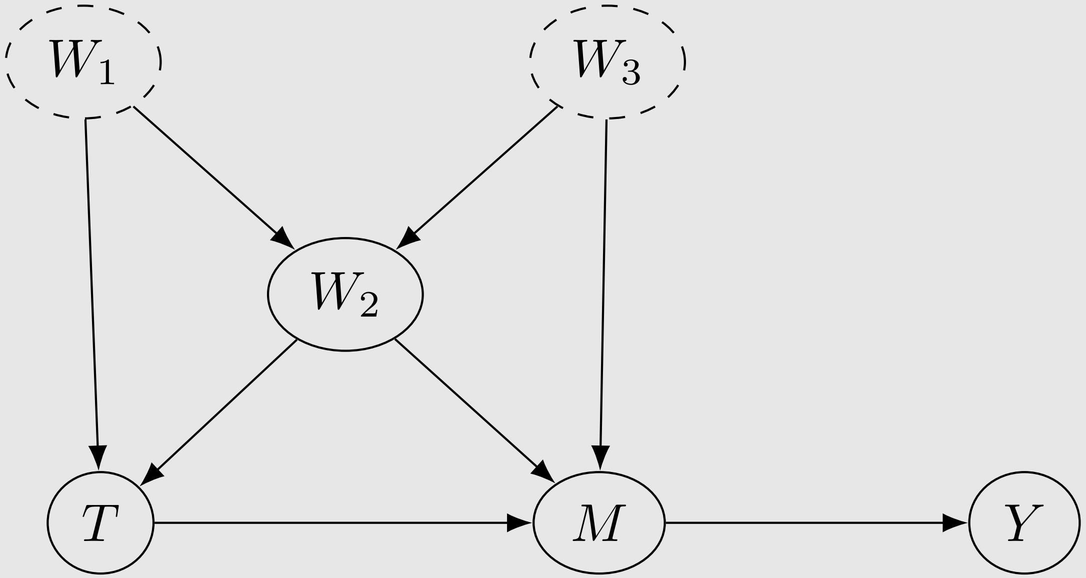




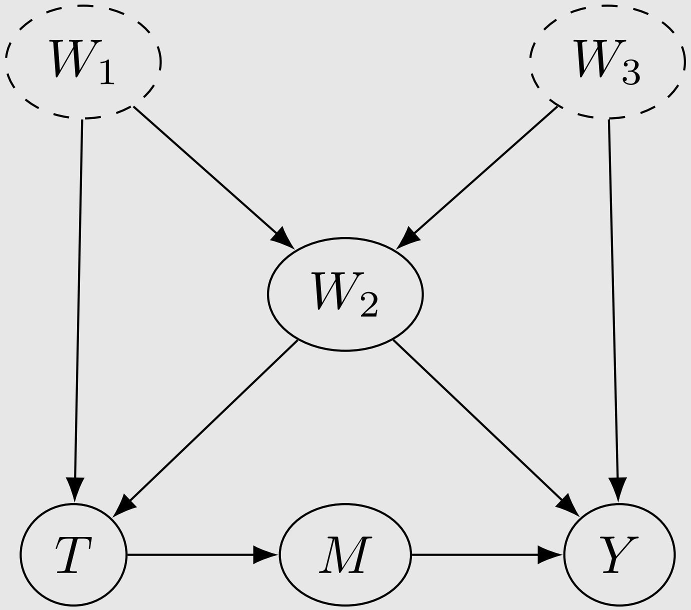
Questions:

1. Is the unconfounded children criterion satisfied here?



Questions:

1. Is the unconfounded children criterion satisfied here?
2. How about here?



Questions:

1. Is the unconfounded children criterion satisfied here?
2. How about here?
3. Can we get identifiability via any simpler criterion that we've seen before?