The Flow of Association and Causation in Graphs

Brady Neal

causalcourse.com

Graph terminology

Bayesian networks and causal graphs

The basic building blocks of graphs

The flow of association and causation

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Graph terminology: not a graph

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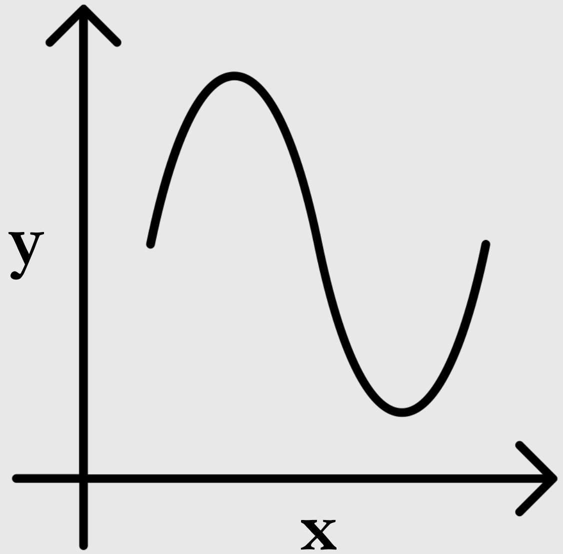

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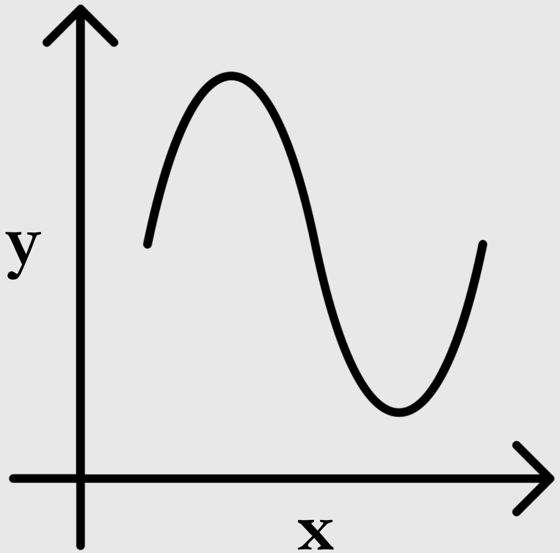

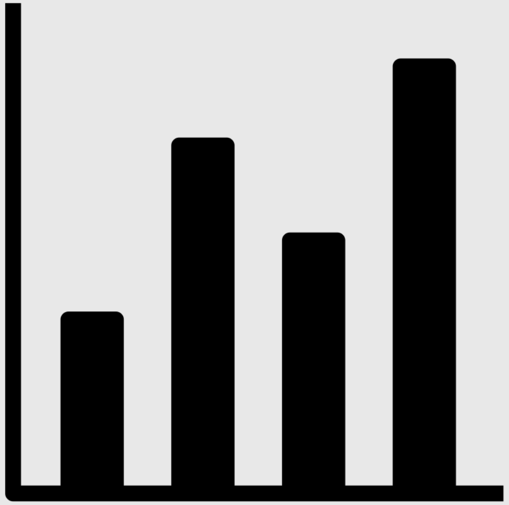

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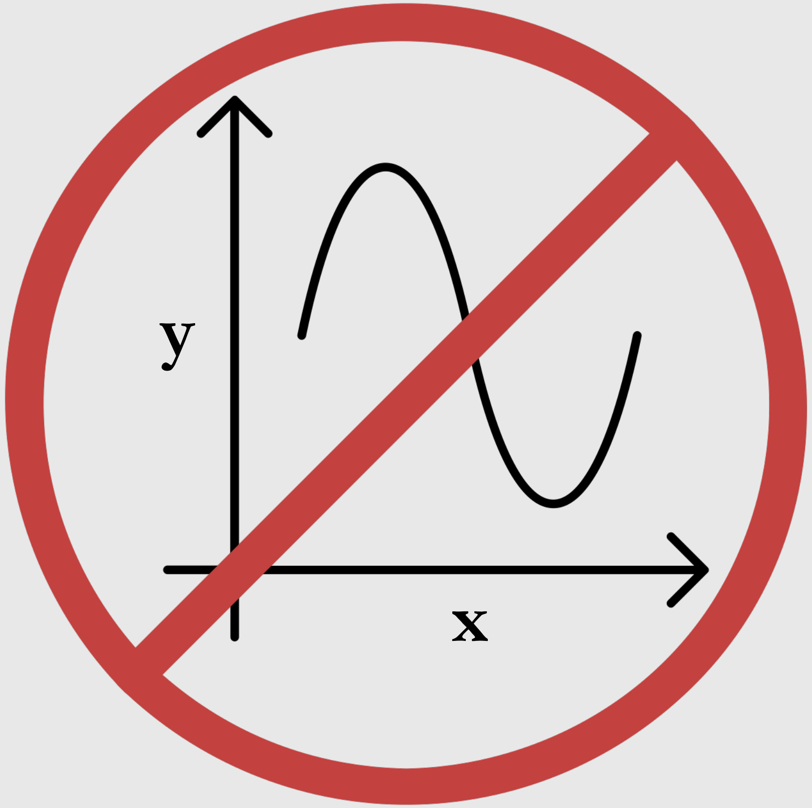
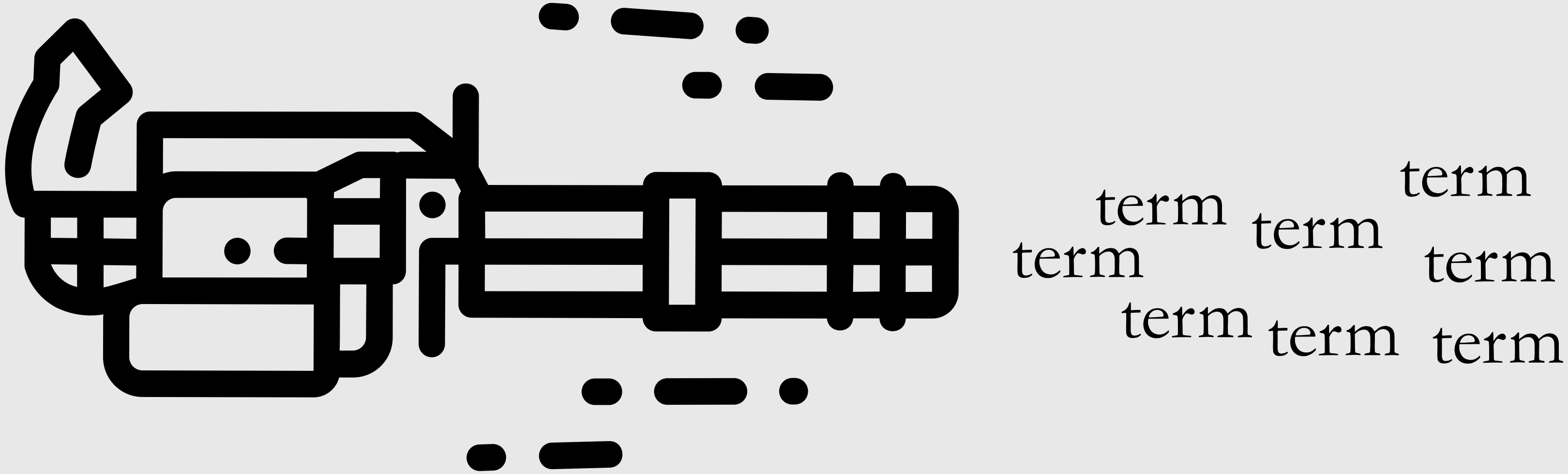


Graph terminology: Terminology Machine Gun



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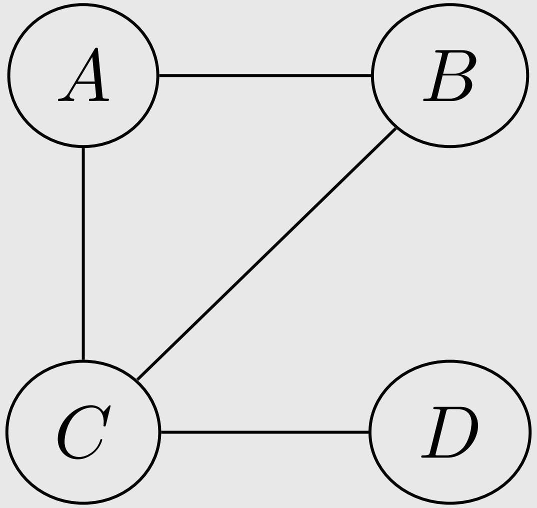
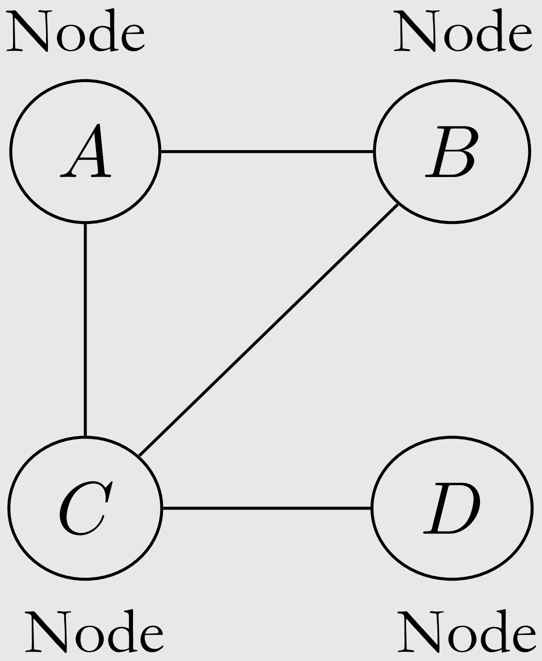


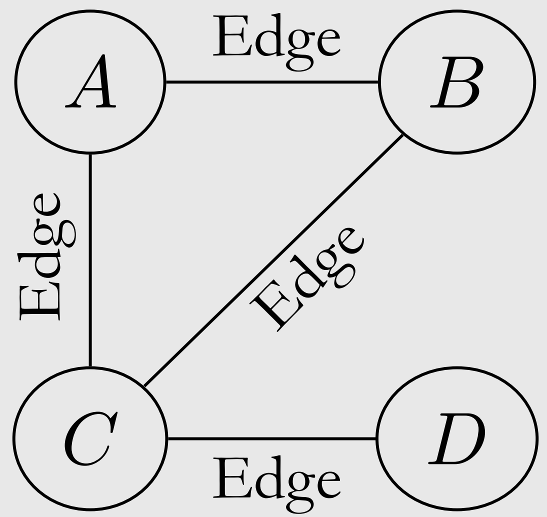
Graph terminology

Nodes



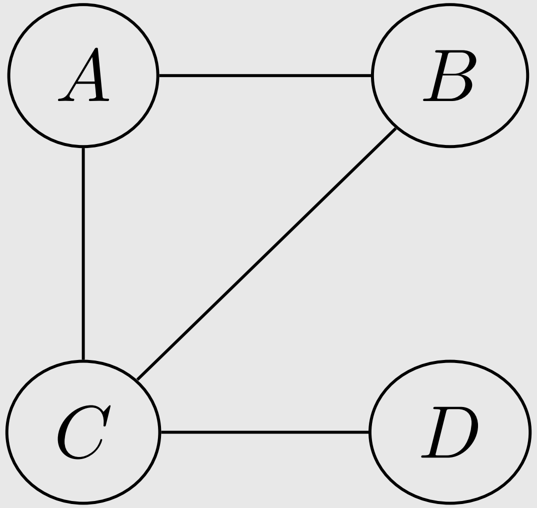
Graph terminology

Edges



Graph terminology

Undirected Graph



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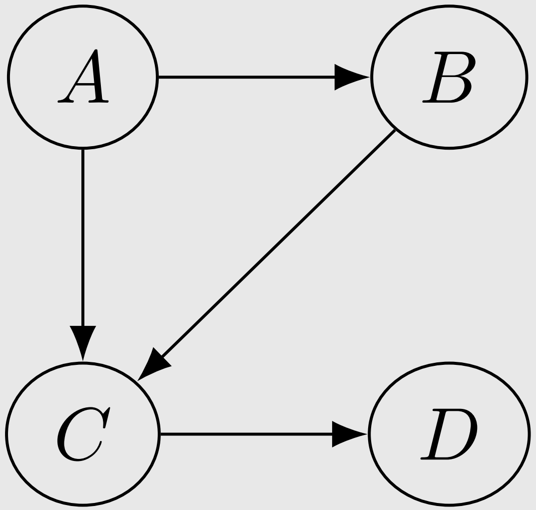

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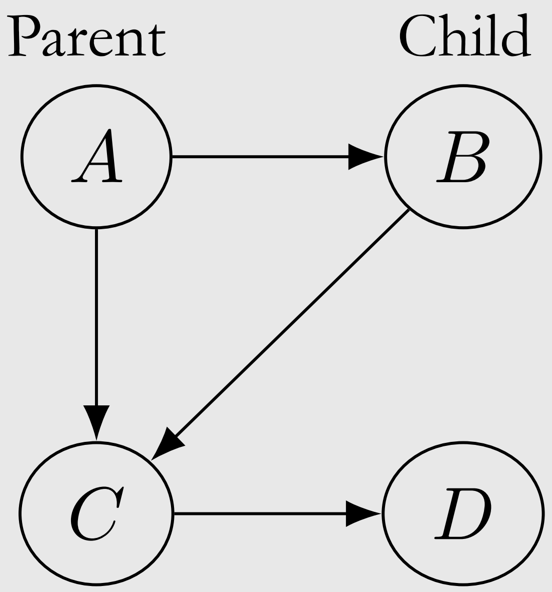
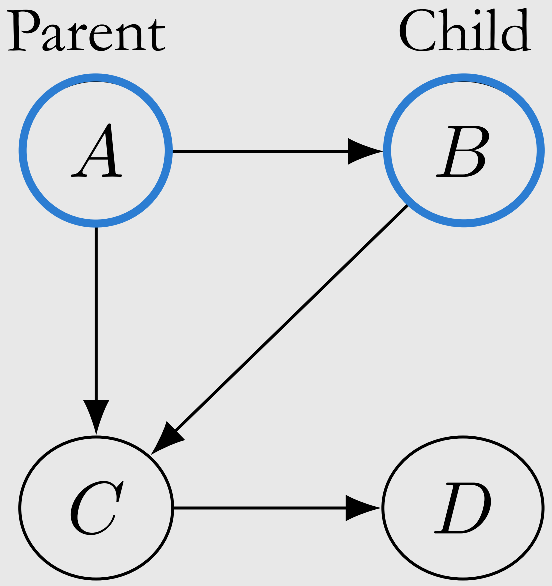


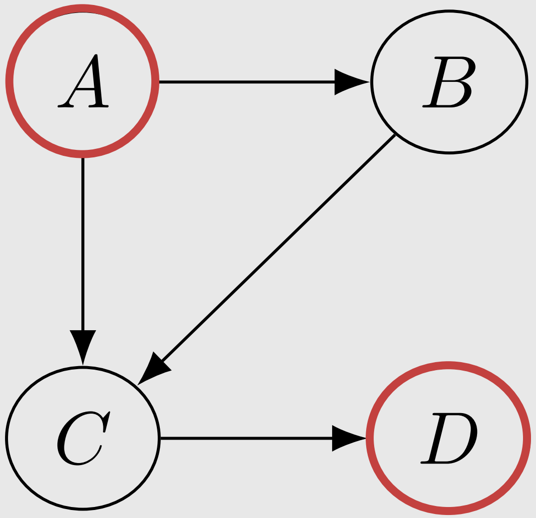
Graph terminology

Adjacent



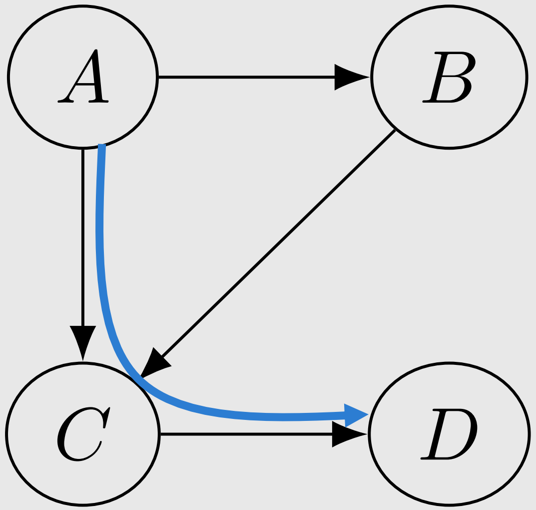
Graph terminology

Not
Adjacent



Graph terminology

Path



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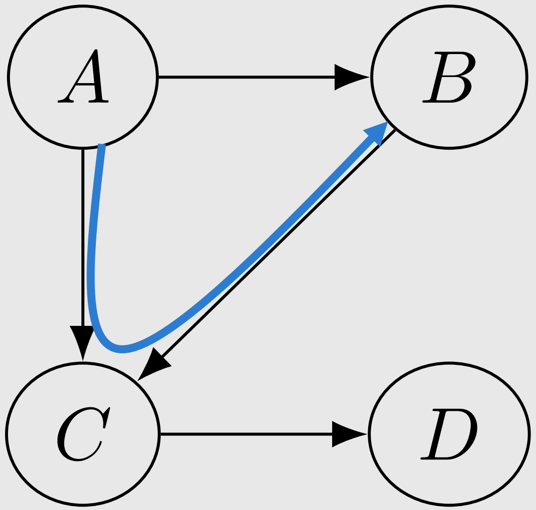

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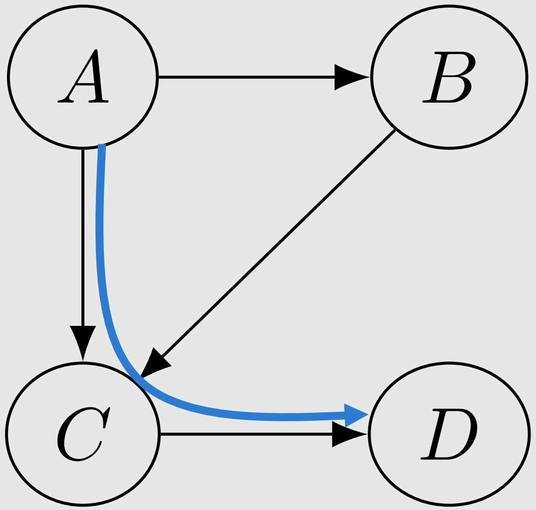

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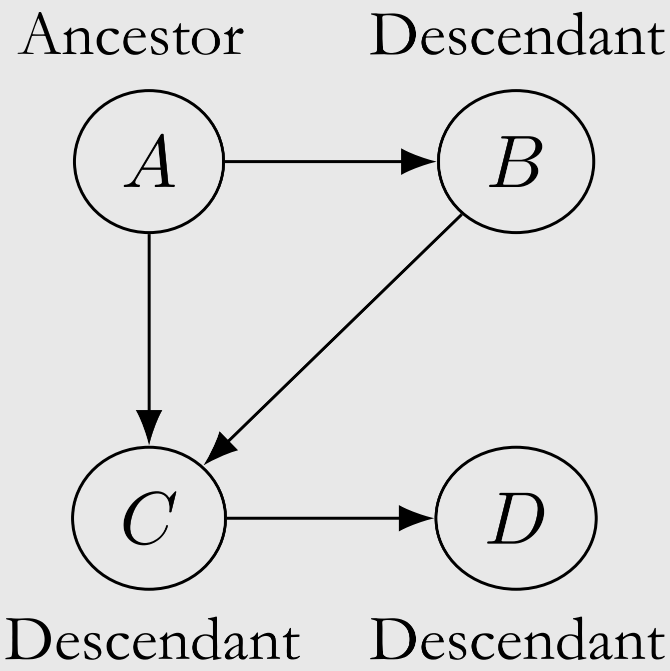

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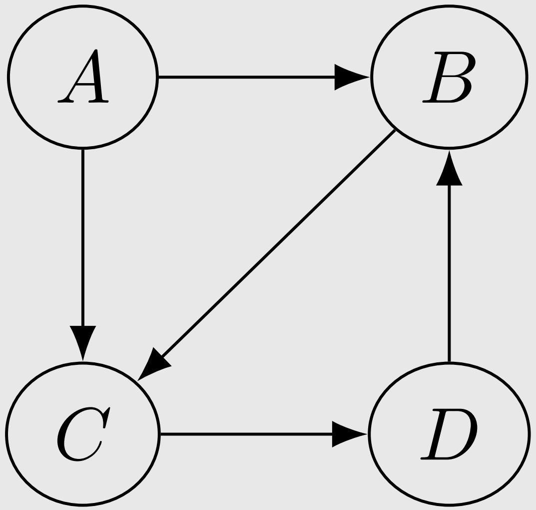

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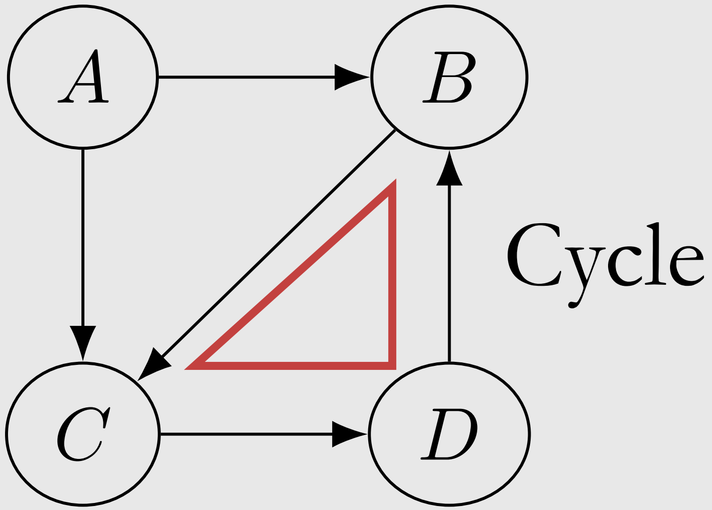

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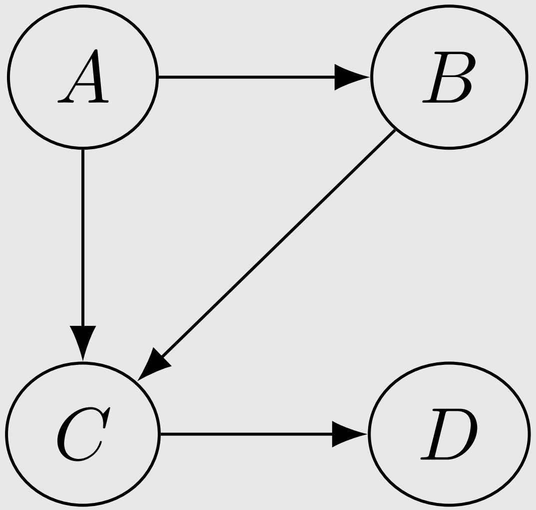

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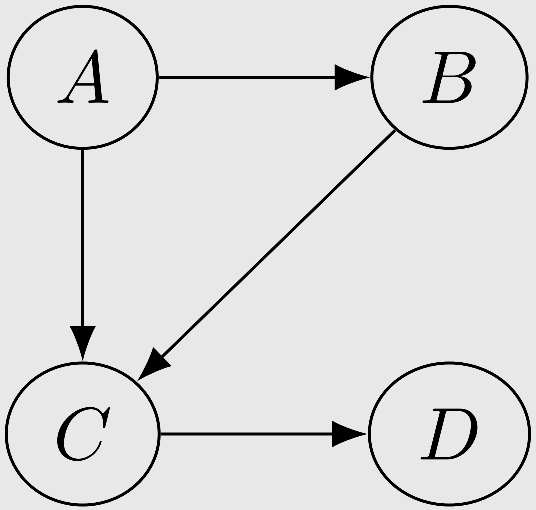
Directed Acyclic Graph (DAG)



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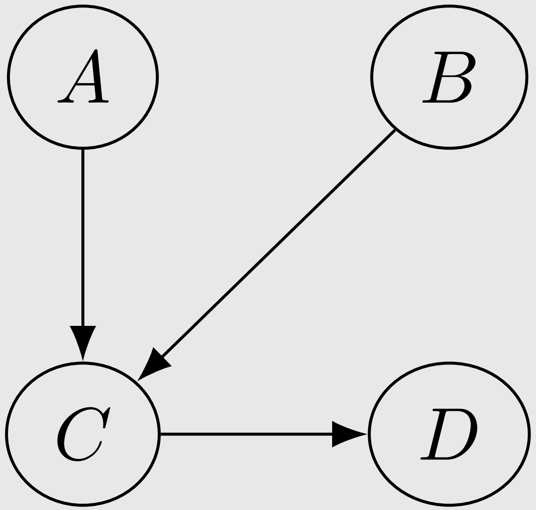

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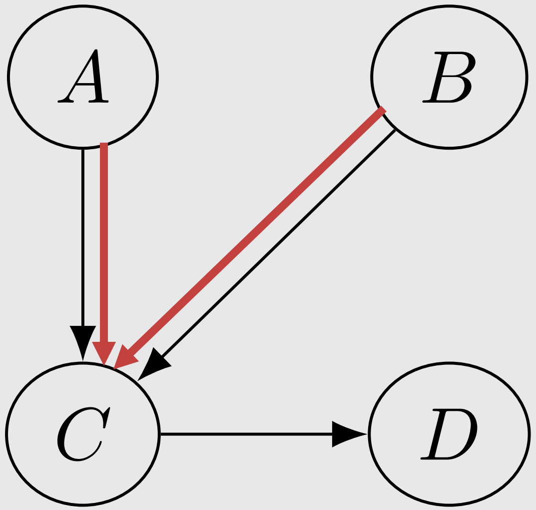
Immorality



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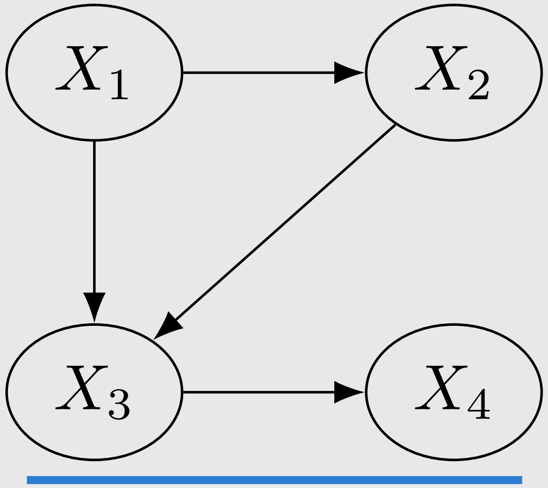

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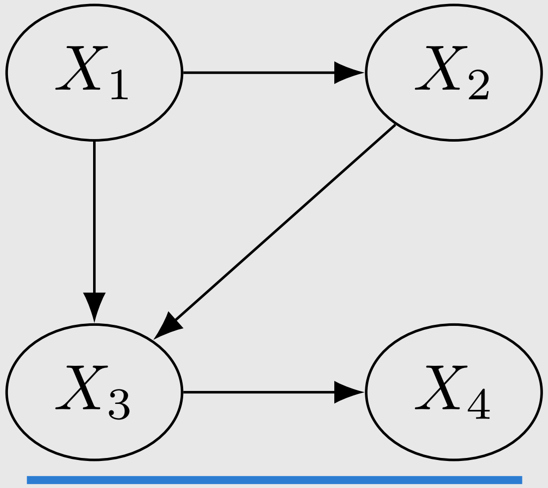

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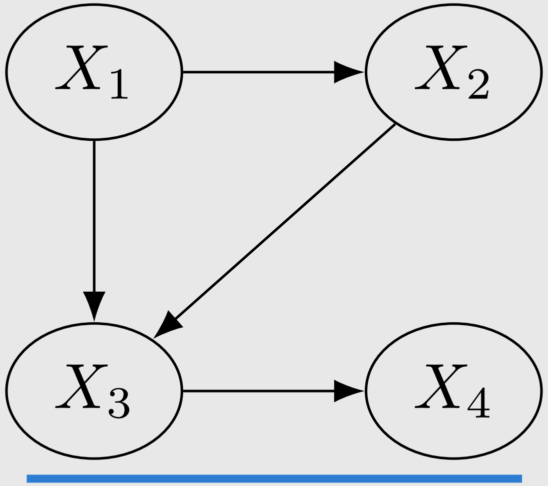

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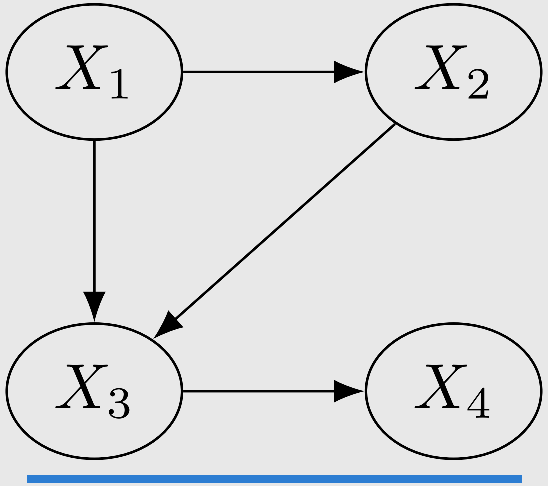

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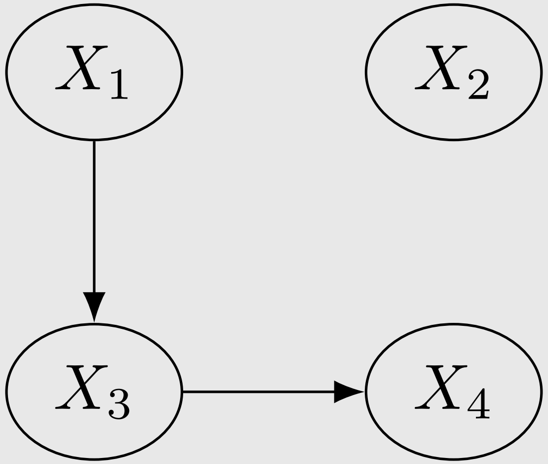

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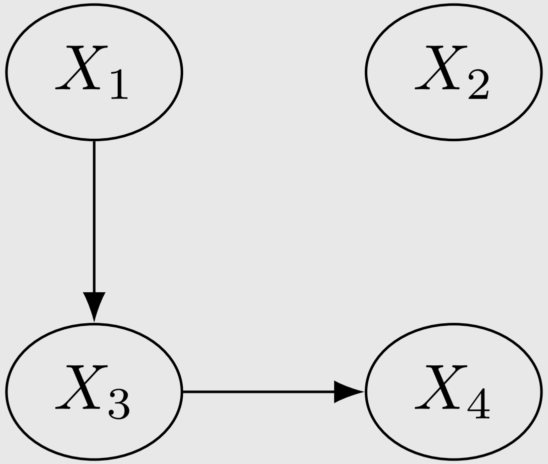
Question:
How will the factorization change now?



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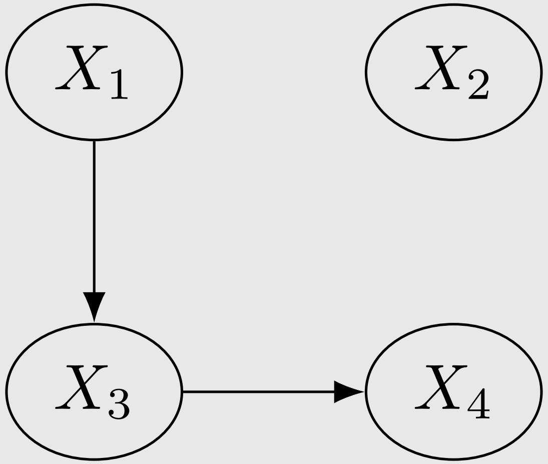

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See Chapter 3 of Koller & Friedman (2009) book for proofs

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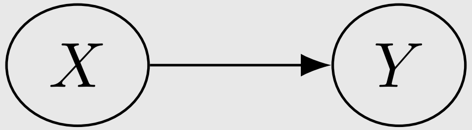

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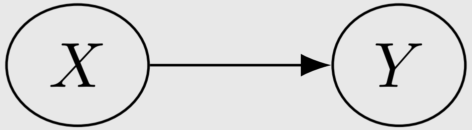

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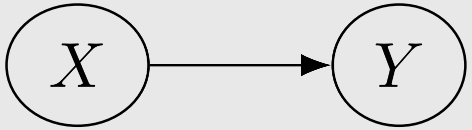

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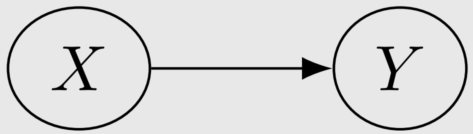
2. Adjacent nodes in the DAG are dependent.



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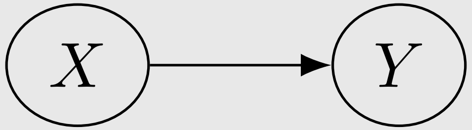

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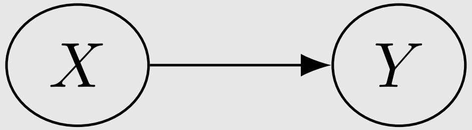

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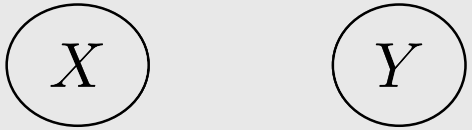
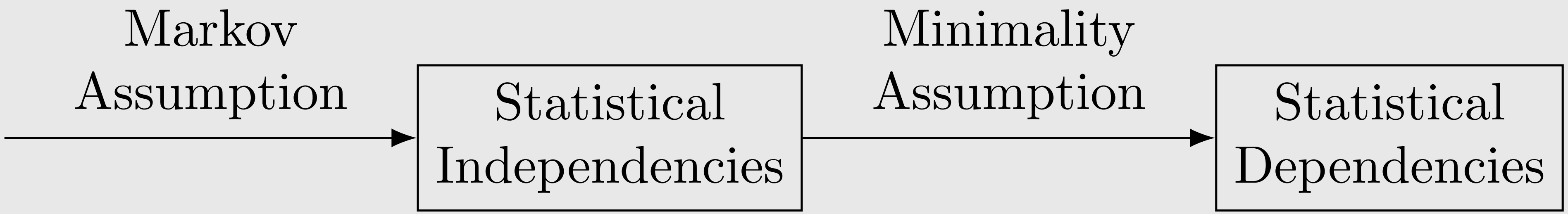


Assumptions flowchart



Recall:

1. How is the local Markov assumption related to the Bayesian network factorization?
2. What are the two parts of the minimality assumption? What do we gain with the second part?

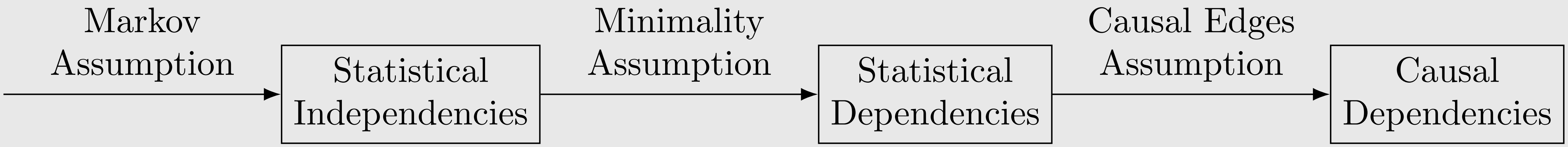
What is a cause?

A variable X is said to be a cause of a variable Y if Y can change in response to changes in X.

Causal edges assumption

In a directed graph, every parent is a direct cause of all its children.

Assumptions flowchart



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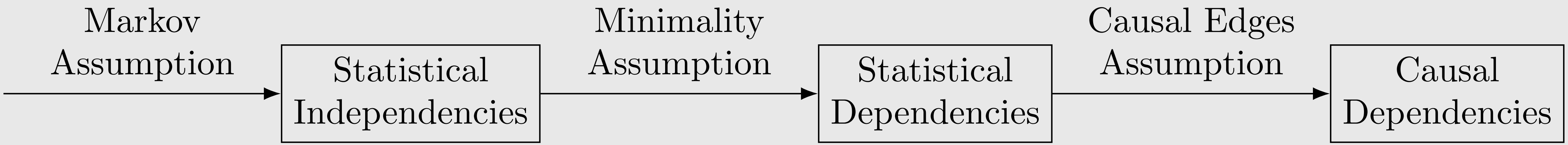

DAG + Two assumptions to give us flow of association and causation in graphs:

1. Markov Assumption
2. Causal Edges Assumption

Graph terminology

Bayesian networks and causal graphs

The basic building blocks of graphs

The flow of association and causation

Graphical building blocks

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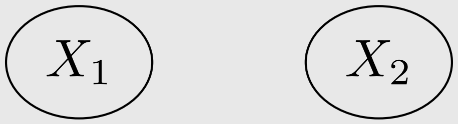
Two nodes:



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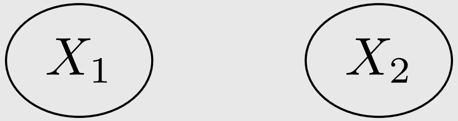
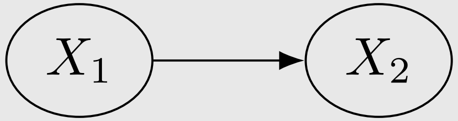
Two nodes: 

or 

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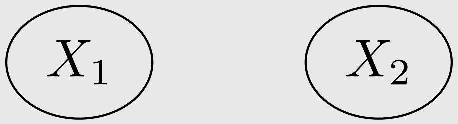
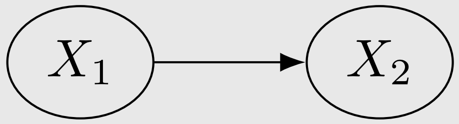
Two nodes: 

or 

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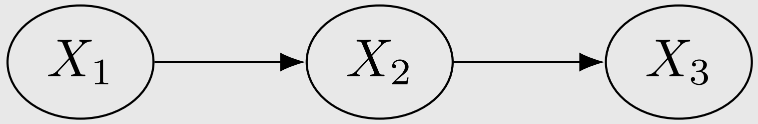

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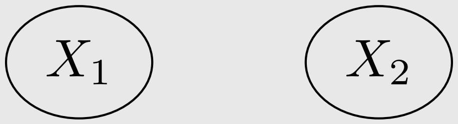

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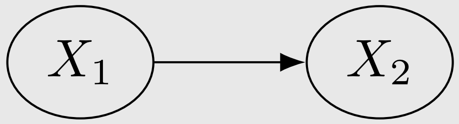

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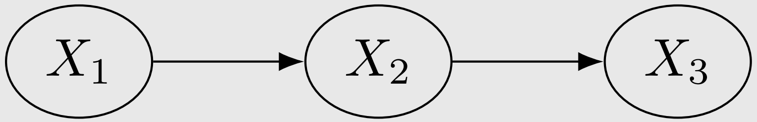
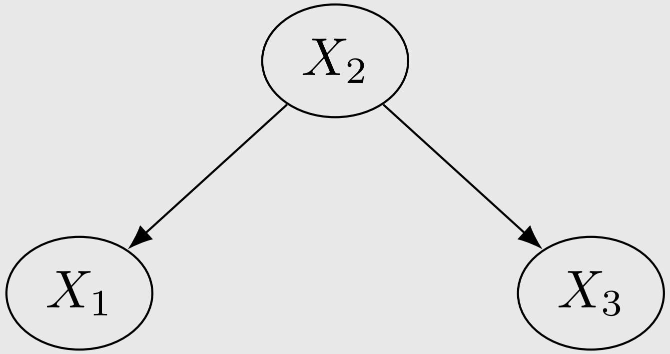
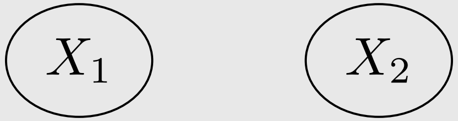


Fork

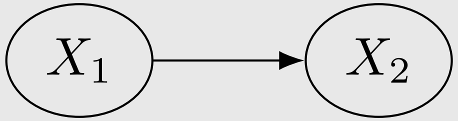


Graphical building blocks

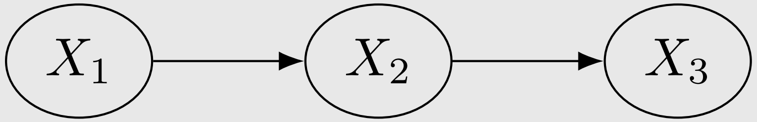
Two nodes:



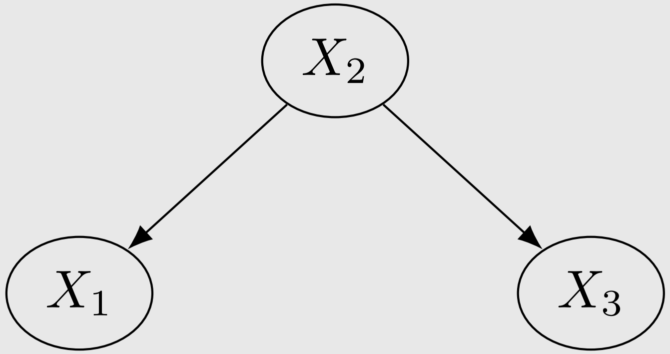
or



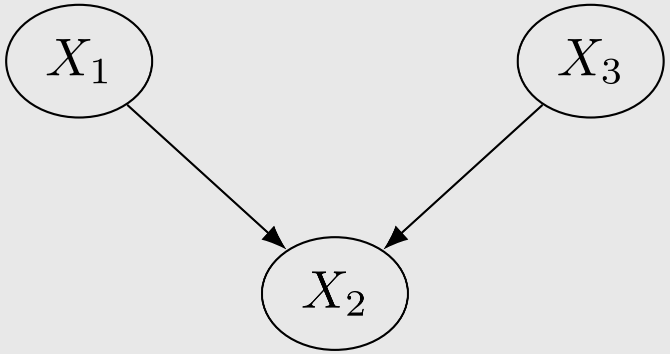
Chain



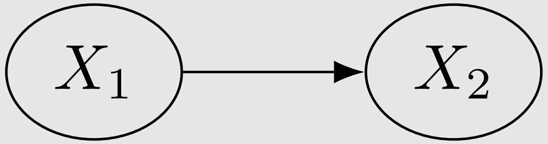
Fork



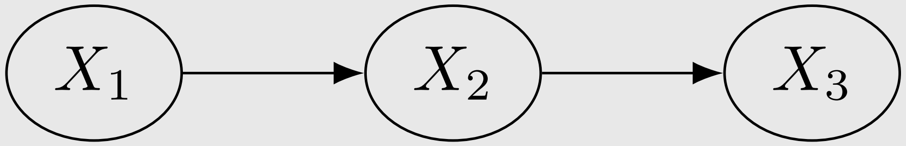
Immorality



Question:
What assumption tells us that X_1 and X_2 are associated, given the following graph?



Chains and forks: dependence



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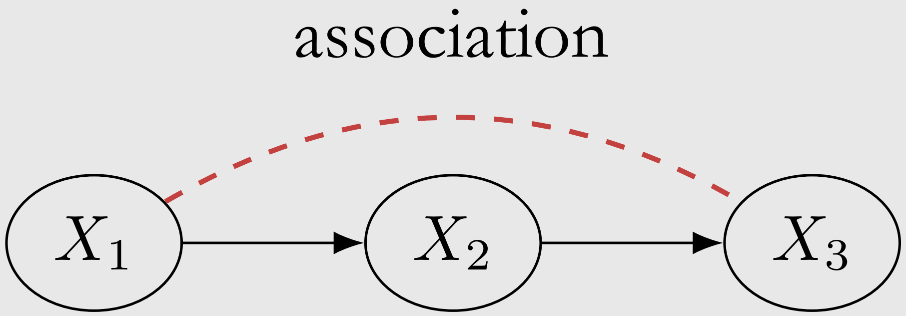

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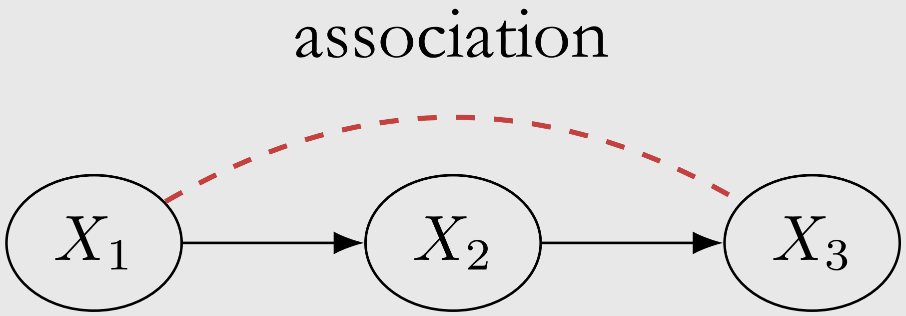

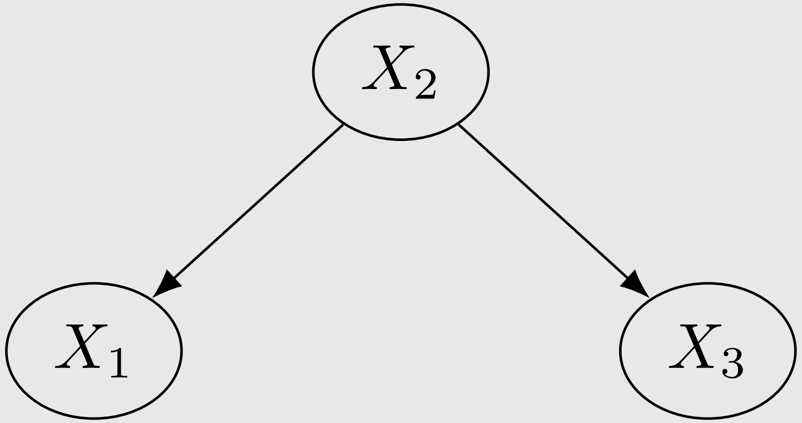

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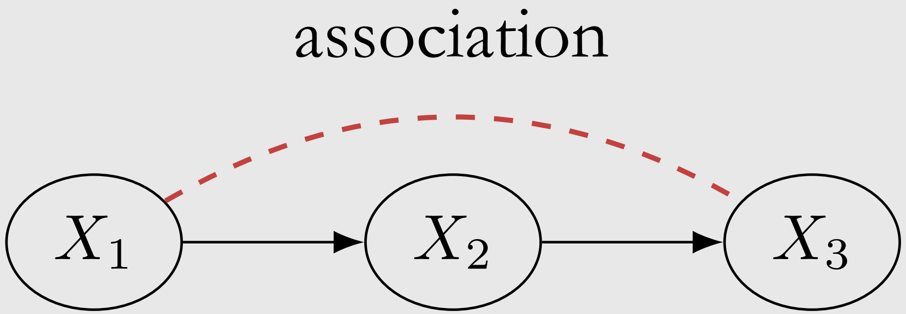

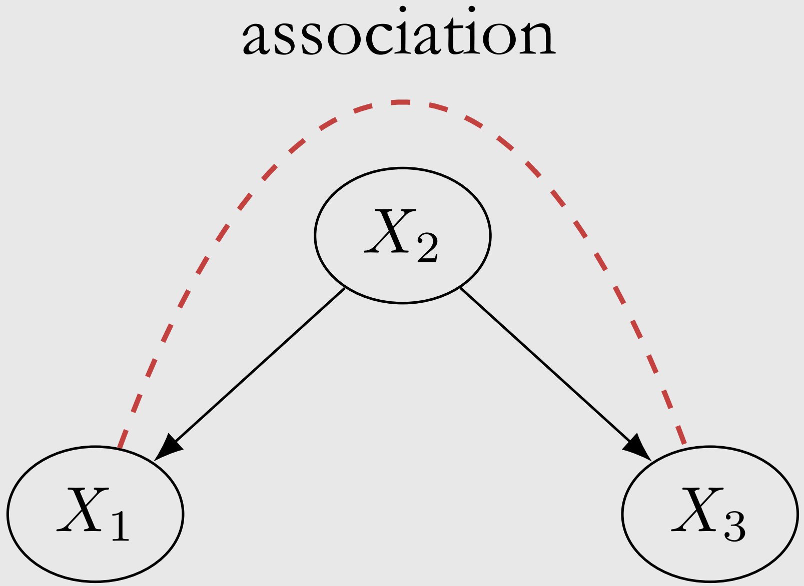

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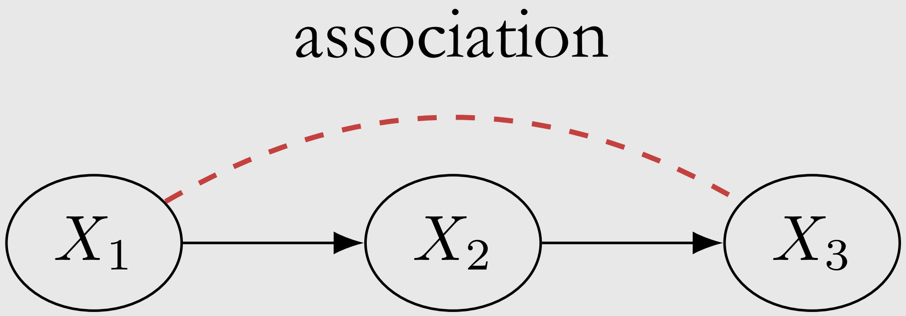

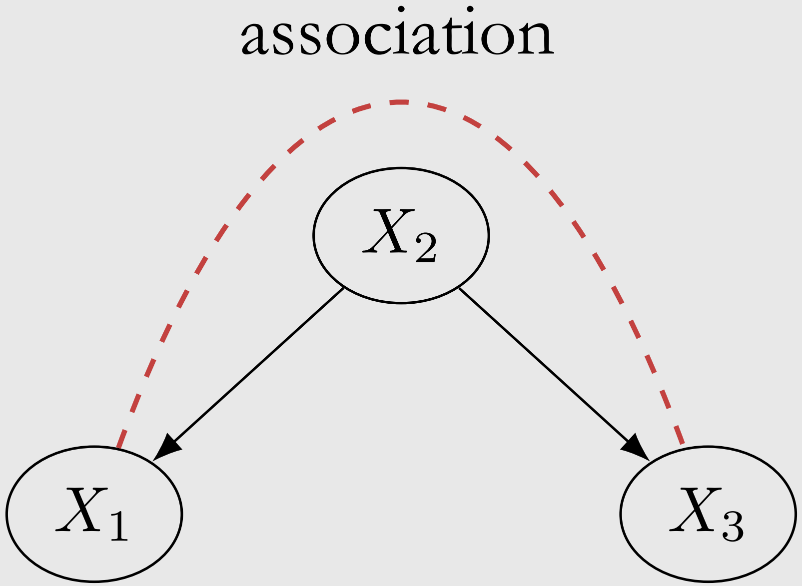

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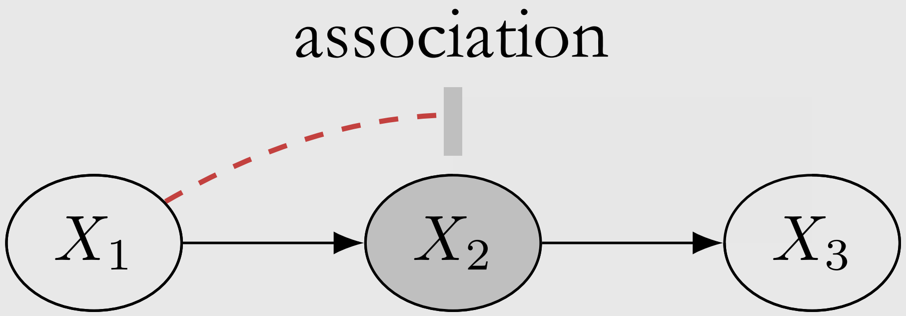

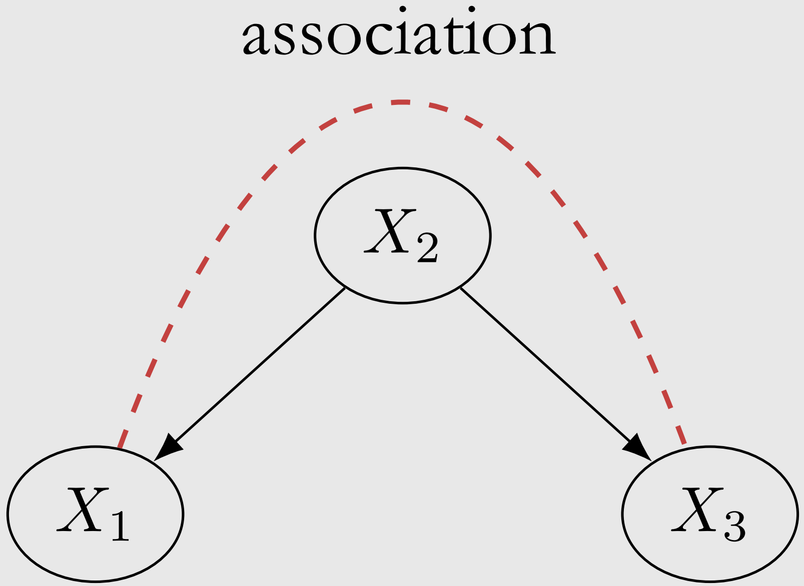

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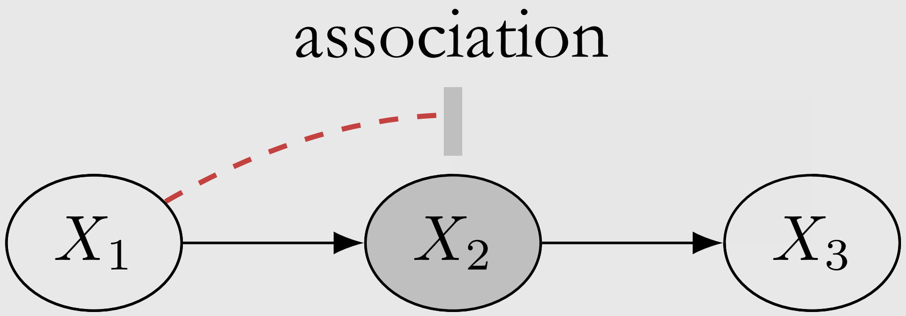

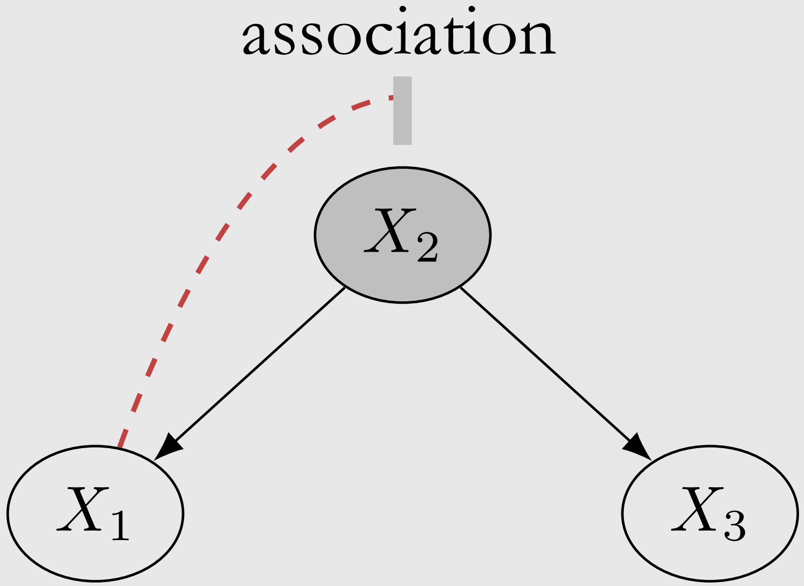

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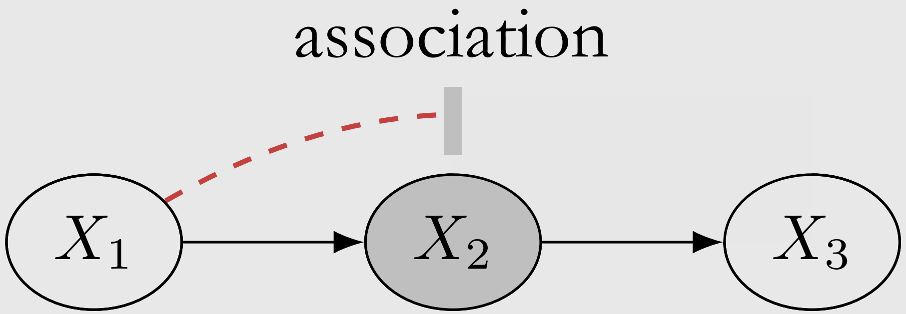
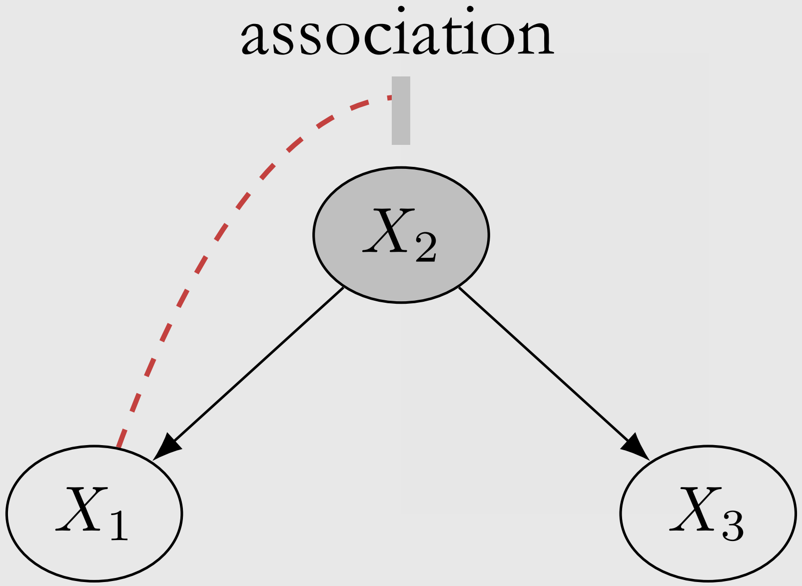
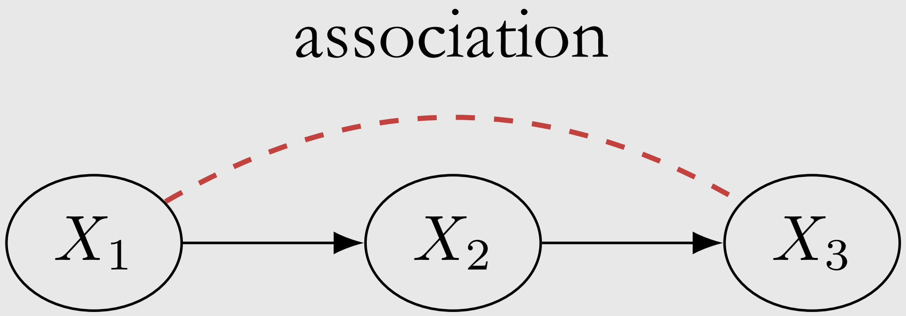


blocked path

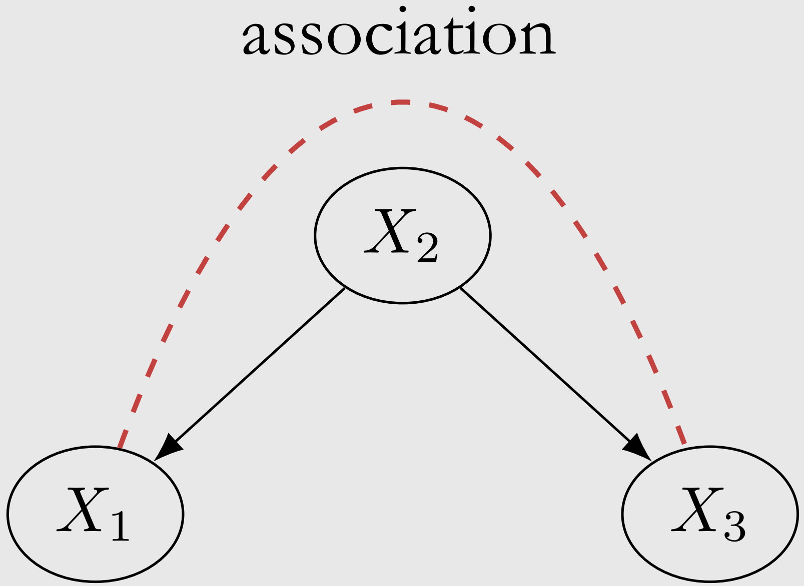


blocked path

Chains and forks: independence

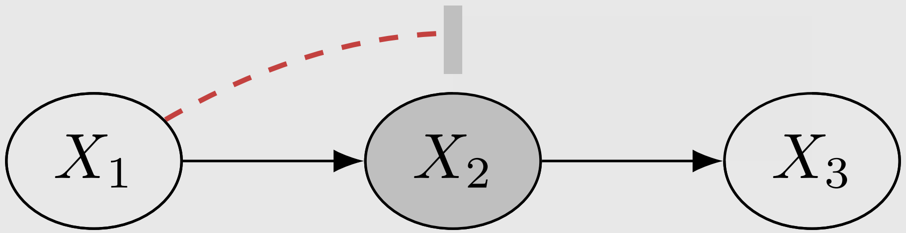


unblocked path



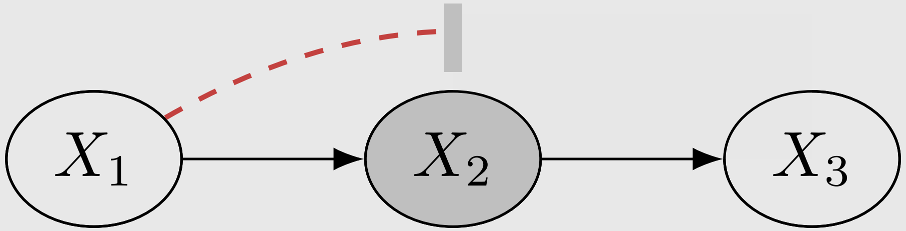
unblocked path

Proof of conditional independence in chains



Proof of conditional independence in chains

Goal: show $P(x_1, x_3 \mid x_2) = P(x_1 \mid x_2)\, P(x_3 \mid x_2)$



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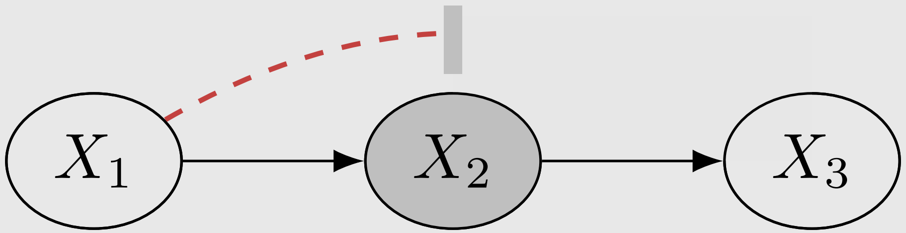

1. Bayesian network factorization:

Proof of conditional independence in chains

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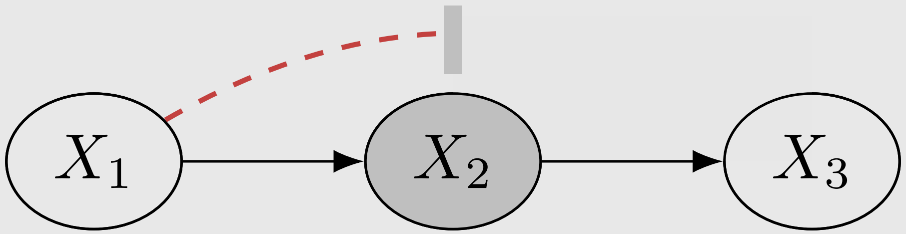

1. Bayesian network factorization: $P(x_1, x_2, x_3) = P(x_1)\, P(x_2|x_1)\, P(x_3|x_2)$

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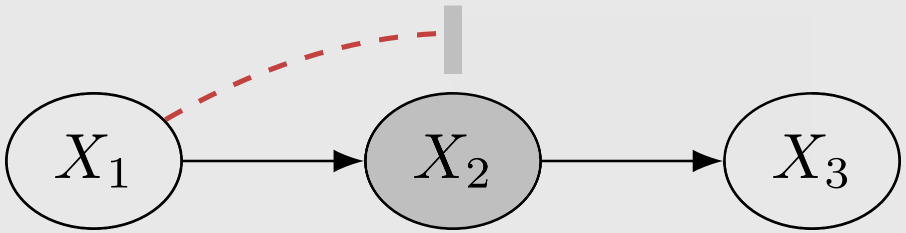

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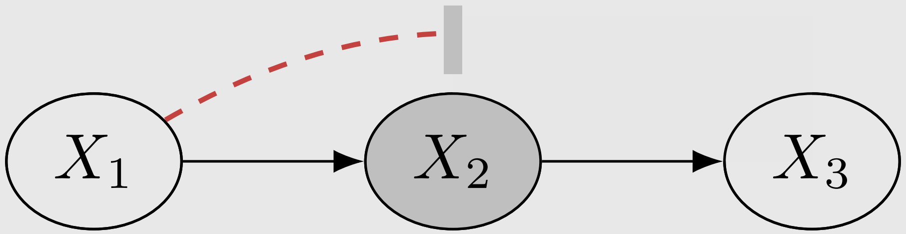

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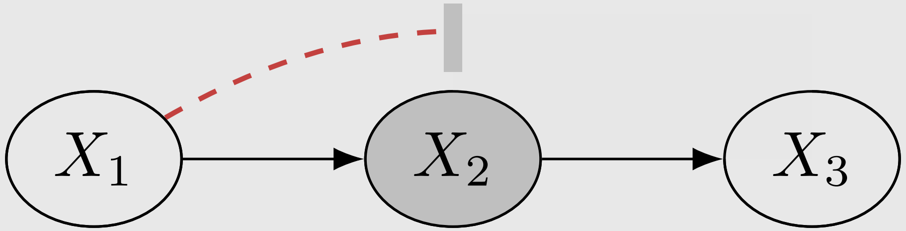

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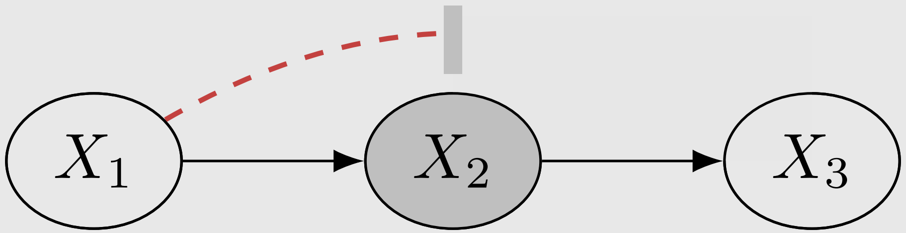
Proof of conditional independence in chains

Goal: show $P(x_1, x_3 \mid x_2) = \boxed{P(x_1 \mid x_2)} P(x_3 \mid x_2)$



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3. Apply Bayes' rule again:

Proof of conditional independence in chains

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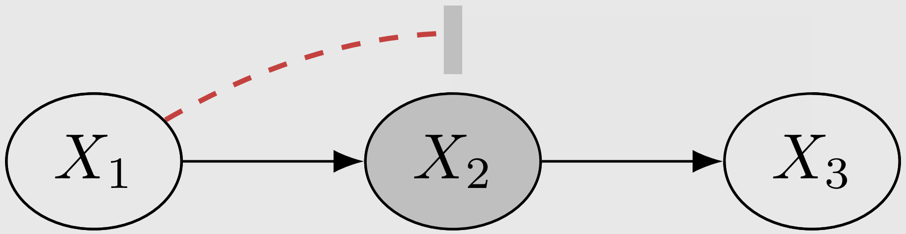

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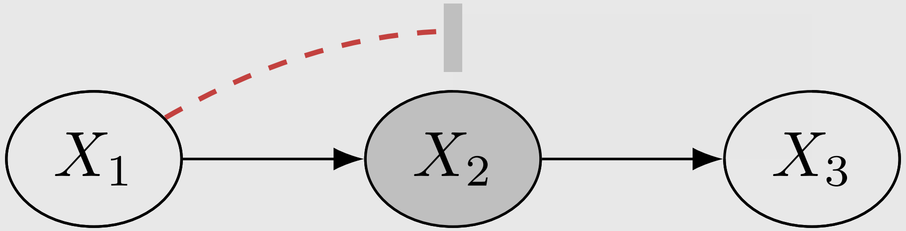

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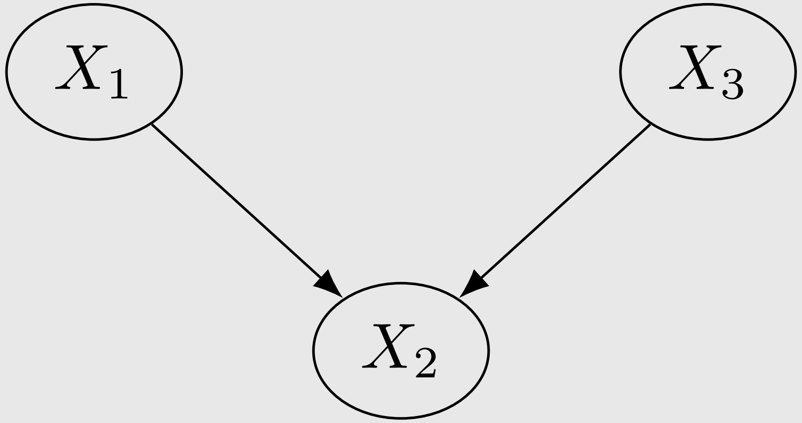
3. Apply Bayes' rule again: $$P(x_1, x_3 \mid x_2) = \frac{P(x_1, x_2)}{P(x_2)}\, P(x_3|x_2)$$ $$= P(x_1|x_2)\, P(x_3|x_2)$$

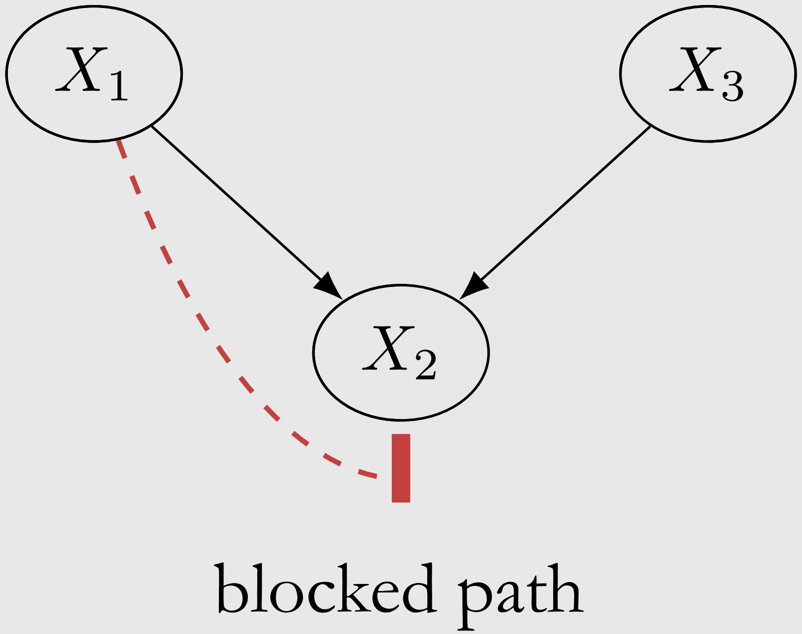
Proof of conditional independence in forks

Your turn ☺

Immoralities



Immoralities



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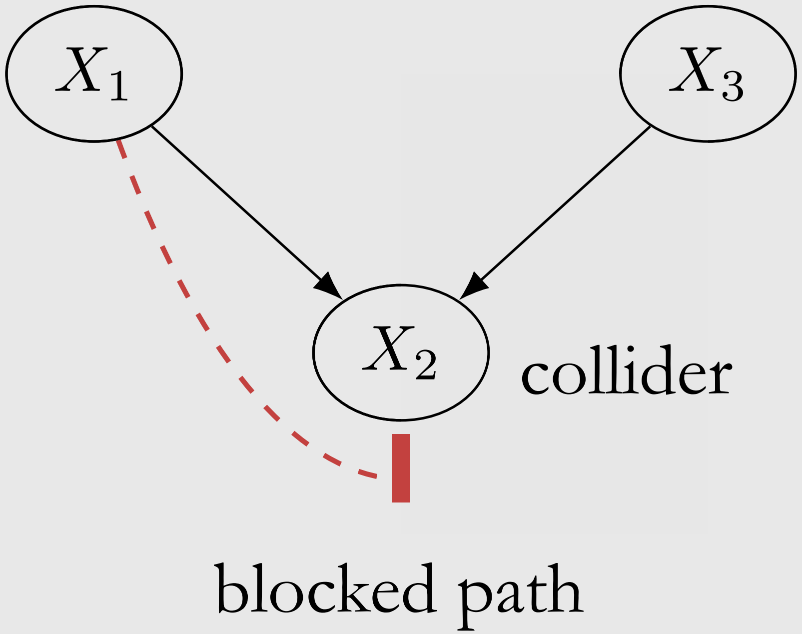
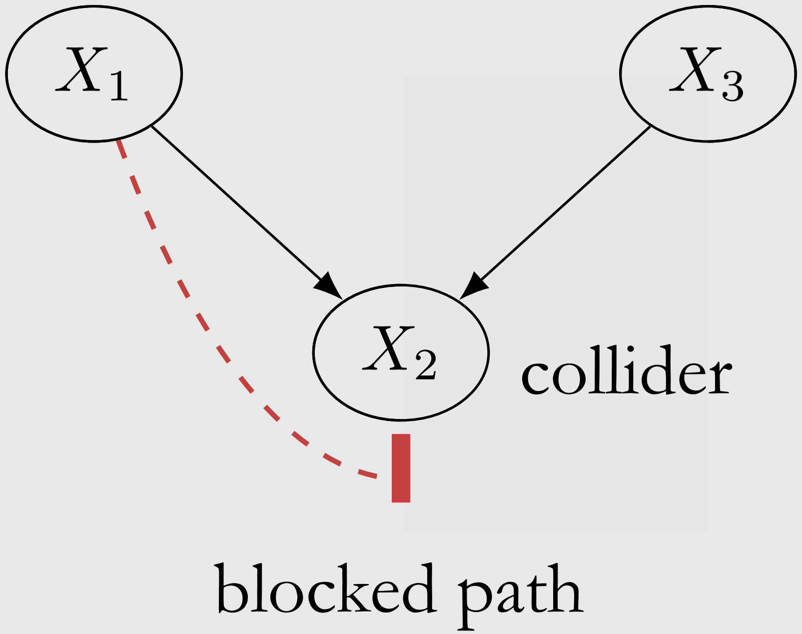
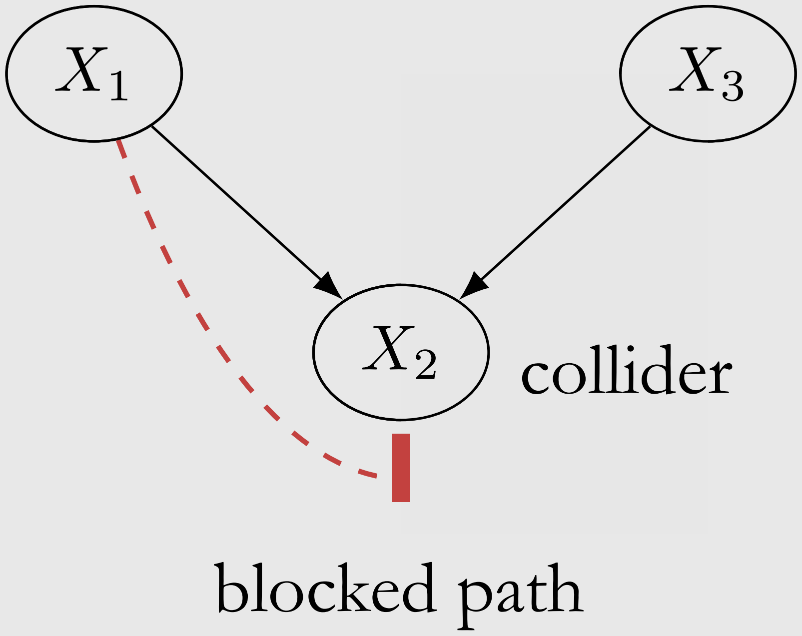


Immoralities



$$P(x_1, x_3) = \sum_{x_2} P(x_1, x_2, x_3)$$

Immoralities



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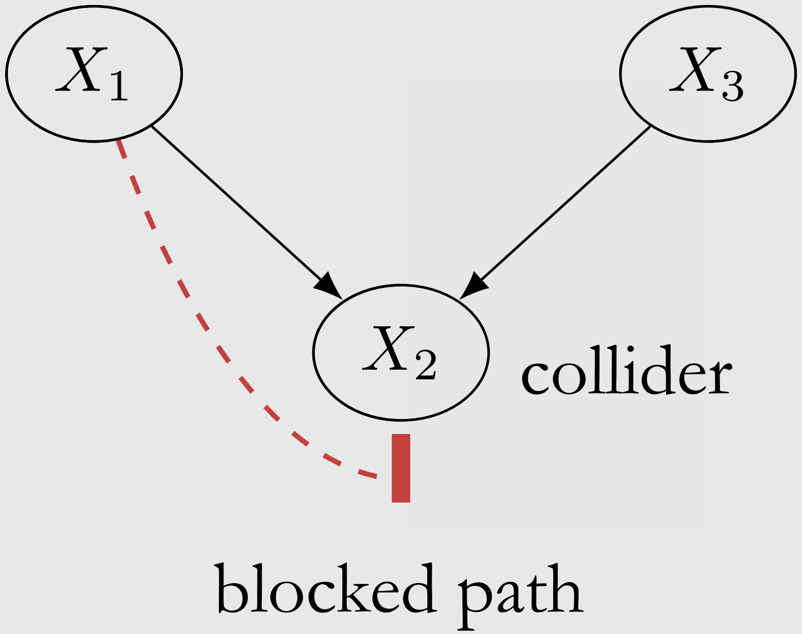

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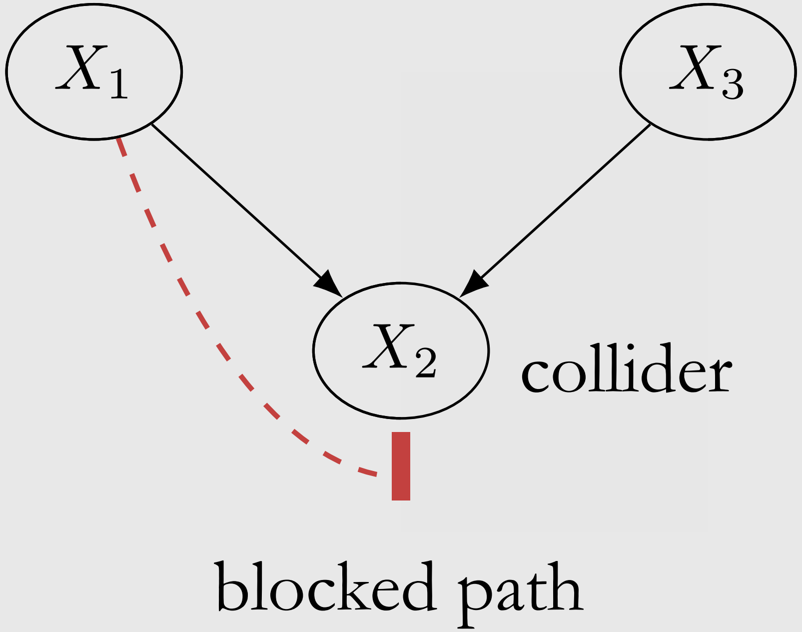

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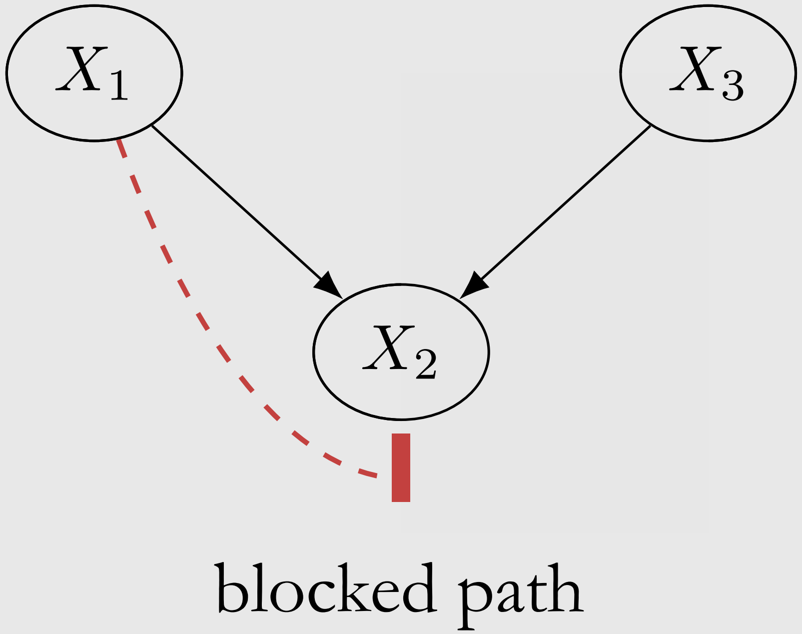
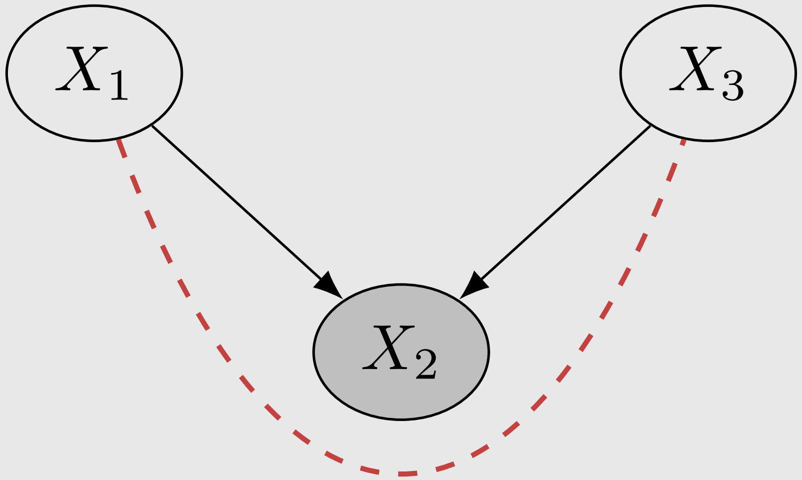
Immoralities: conditioning on the collider

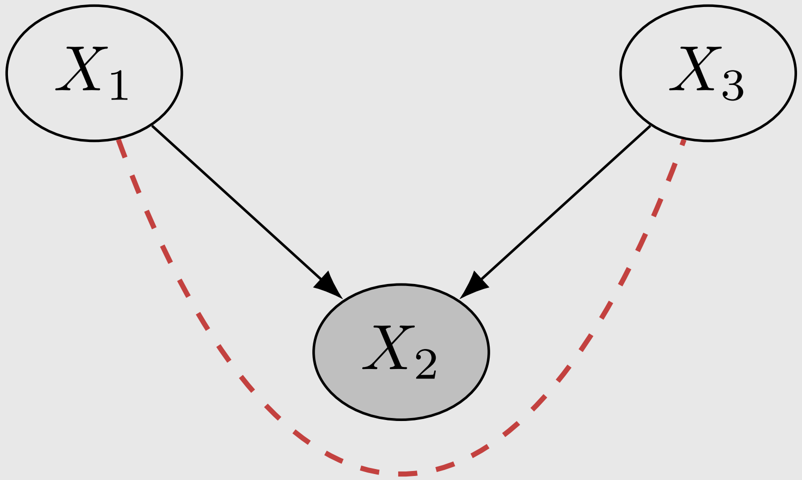


Immoralities: conditioning on the collider



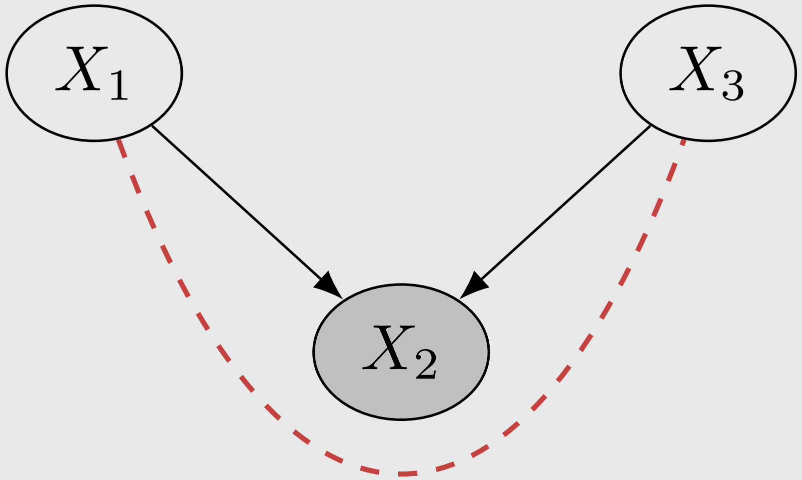
unblocked path

Example: good-looking men are jerks



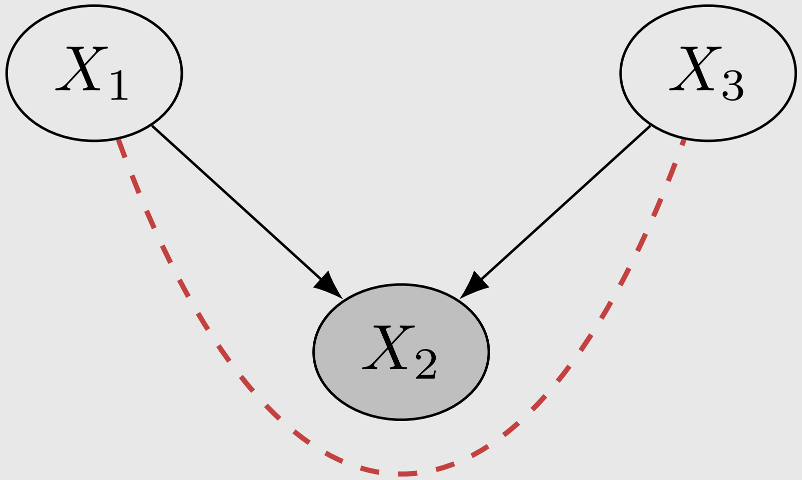
Example: good-looking men are jerks

$$X_1 = \begin{cases} 1 & \text{good-looking} \\ 0 & \text{otherwise} \end{cases}$$



Example: good-looking men are jerks

$$X_1 = \begin{cases} 1 & \text{good-looking} \\ 0 & \text{otherwise} \end{cases} \qquad X_3 = \begin{cases} 1 & \text{kind} \\ 0 & \text{jerk} \end{cases}$$



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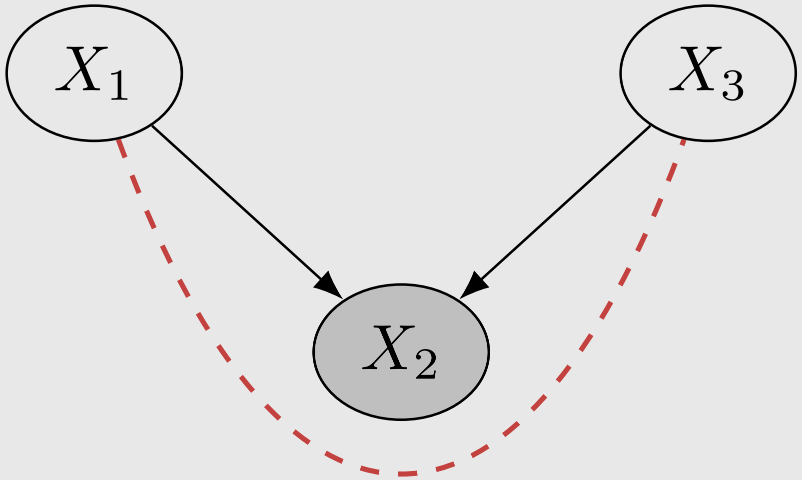

$$X_2 = \begin{cases} 1 & \text{in relationship} \\ 0 & \text{not in relationship} \end{cases}$$

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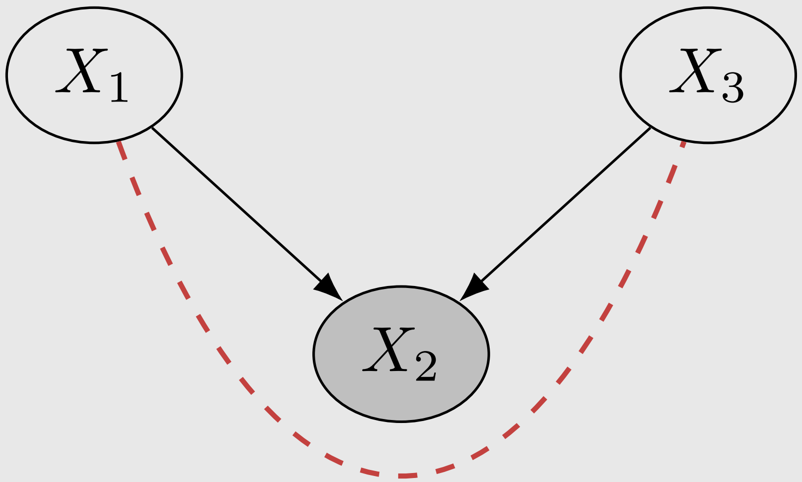
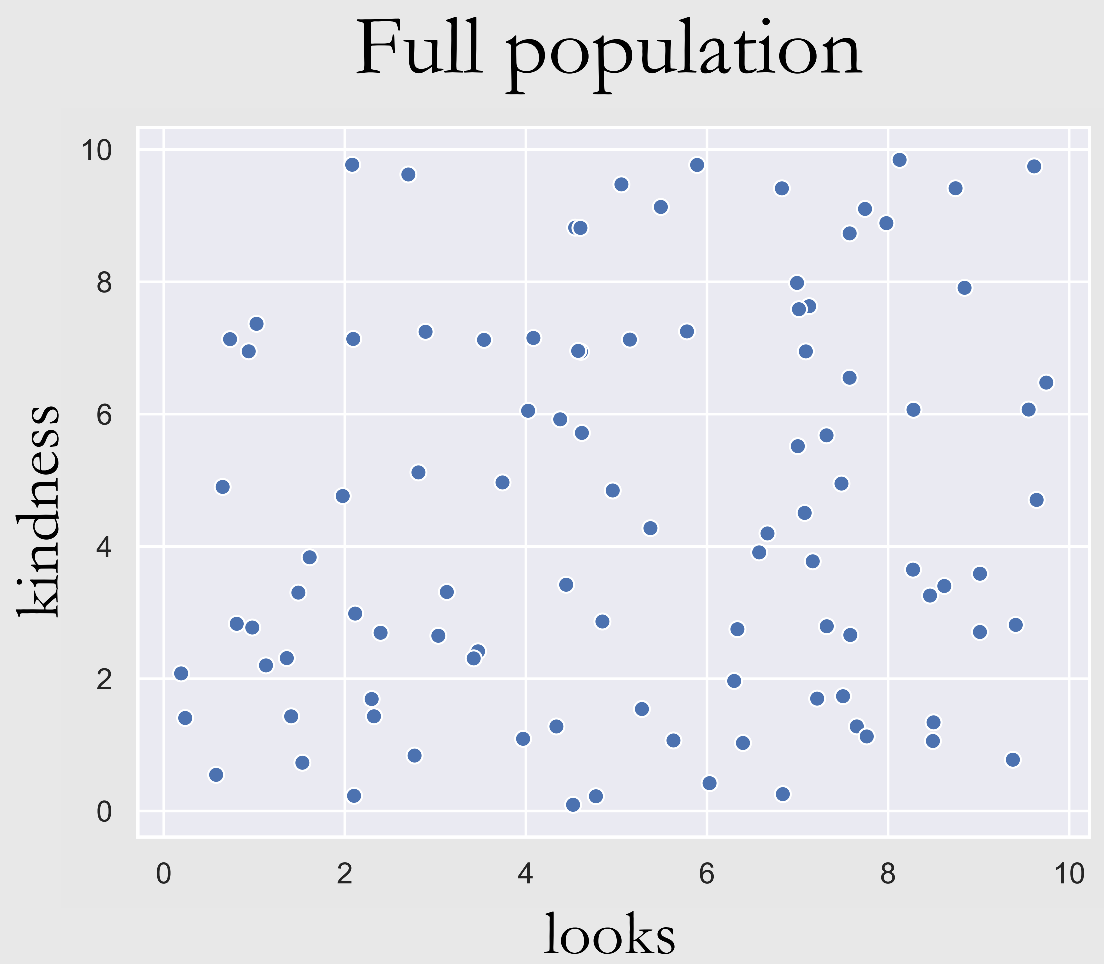


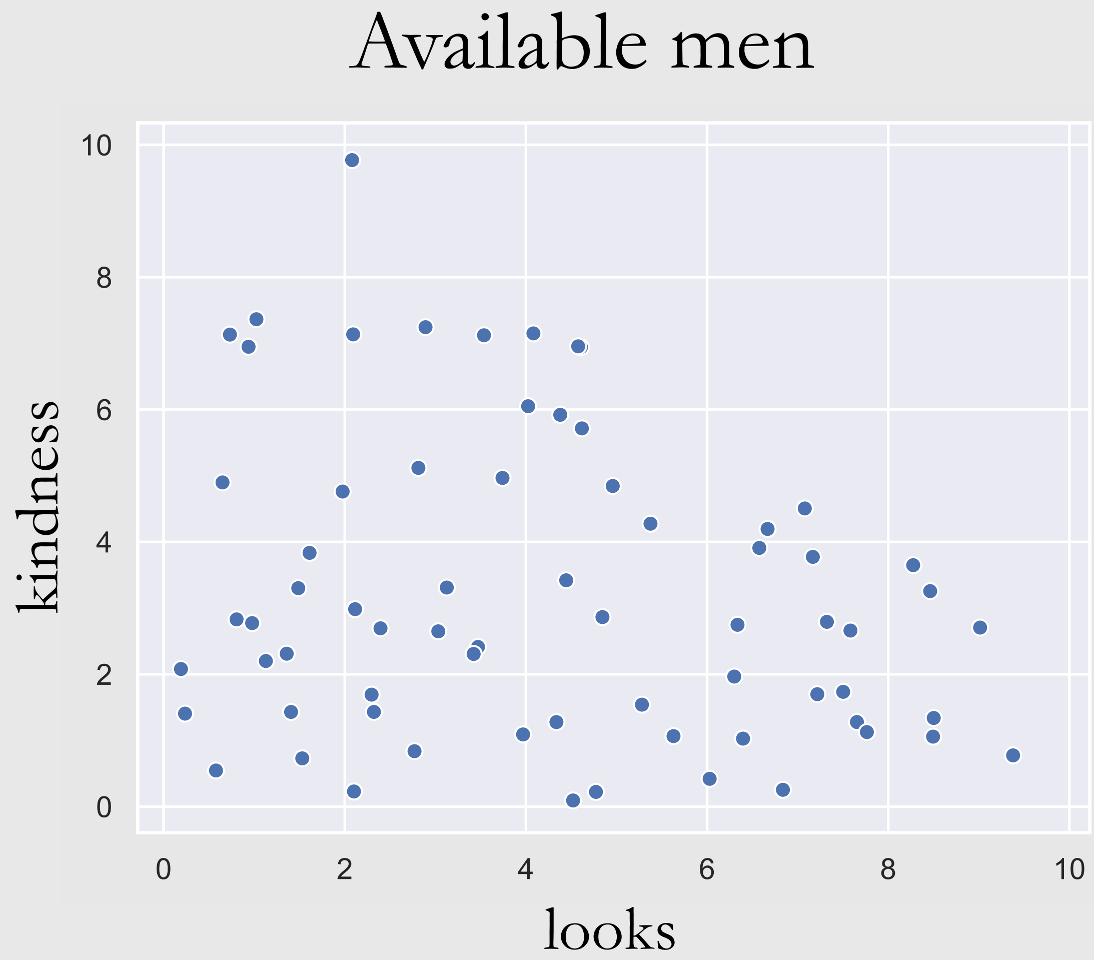
$$X_2 = X_1 \text{ AND } X_3 = \begin{cases} 1 & \text{in relationship} \\ 0 & \text{not in relationship} \end{cases}$$

Good-looking men are jerks scatterplot

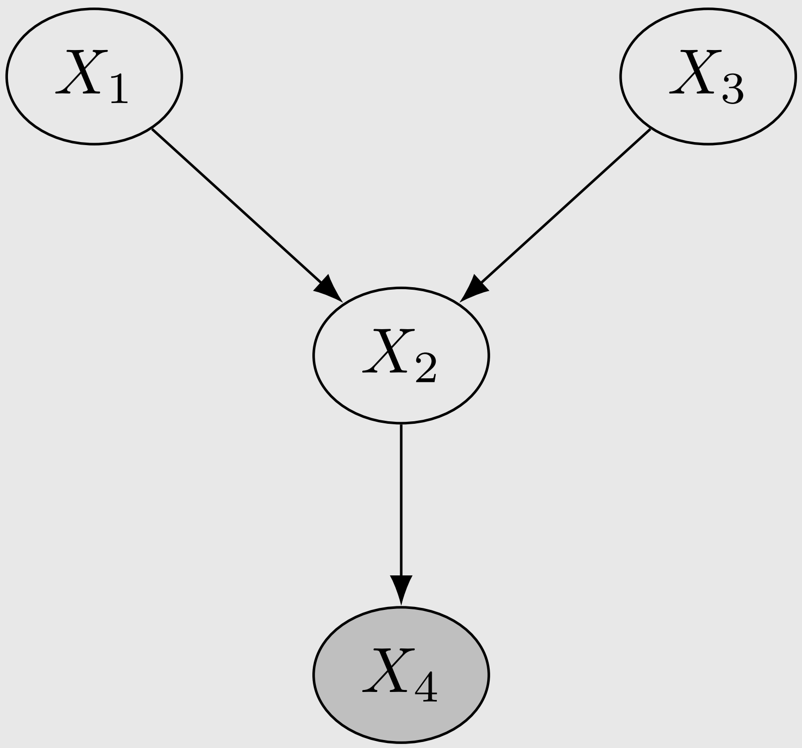


Good-looking men are jerks scatterplot

Good-looking men are jerks scatterplot

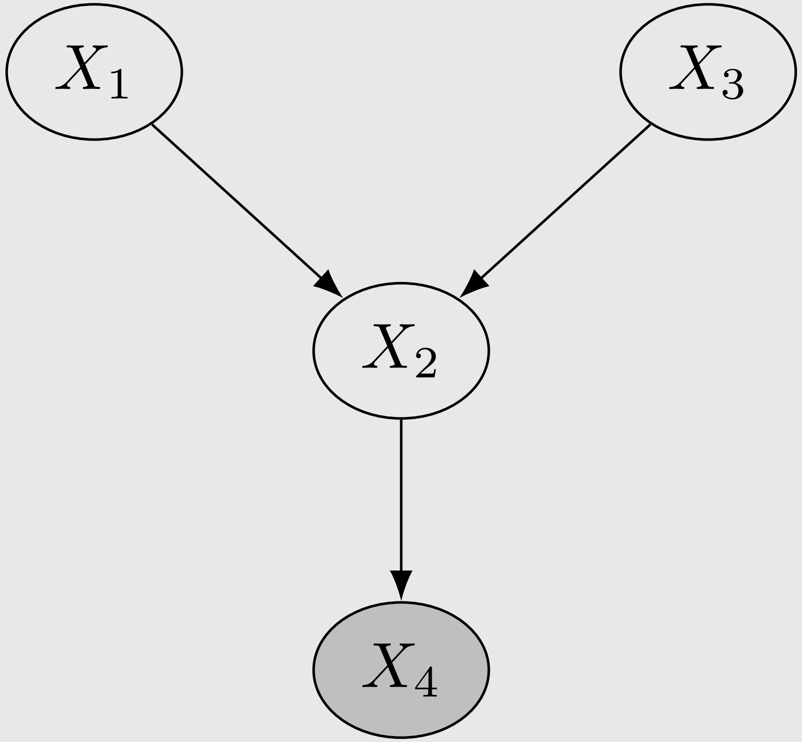


Conditioning on descendants of colliders



Conditioning on descendants of colliders

$$X_1 \not\perp\!\!\!\perp X_3 \mid X_4$$



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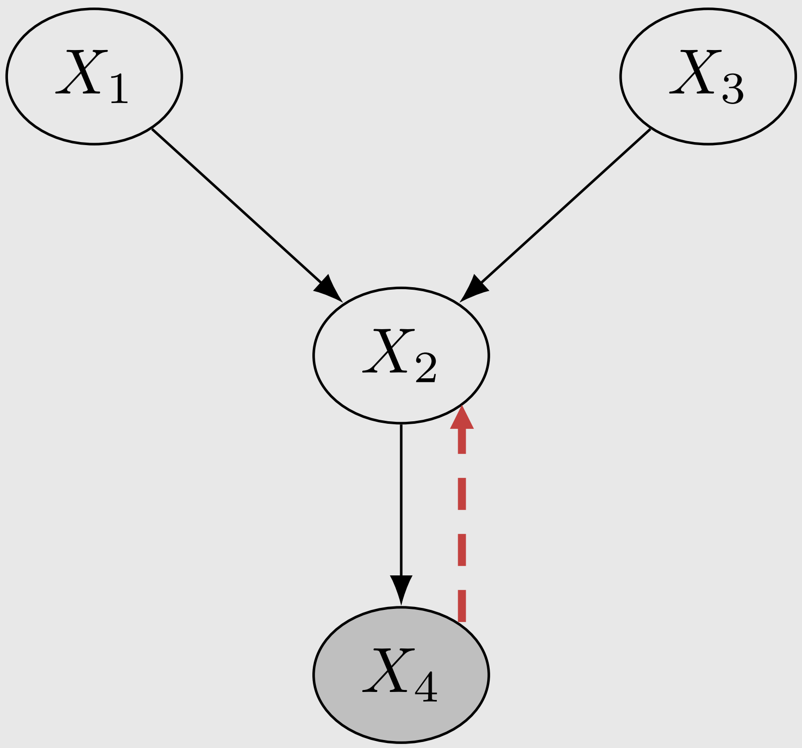

Question:
In the three different kinds of three-node graphs, what can block a path?

Graph terminology

Bayesian networks and causal graphs

The basic building blocks of graphs

The flow of association and causation

Blocked path definition

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A path between nodes X and Y is blocked by a (potentially empty) conditioning set Z if either of the following is true:

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Unblocked path: a path that is not blocked

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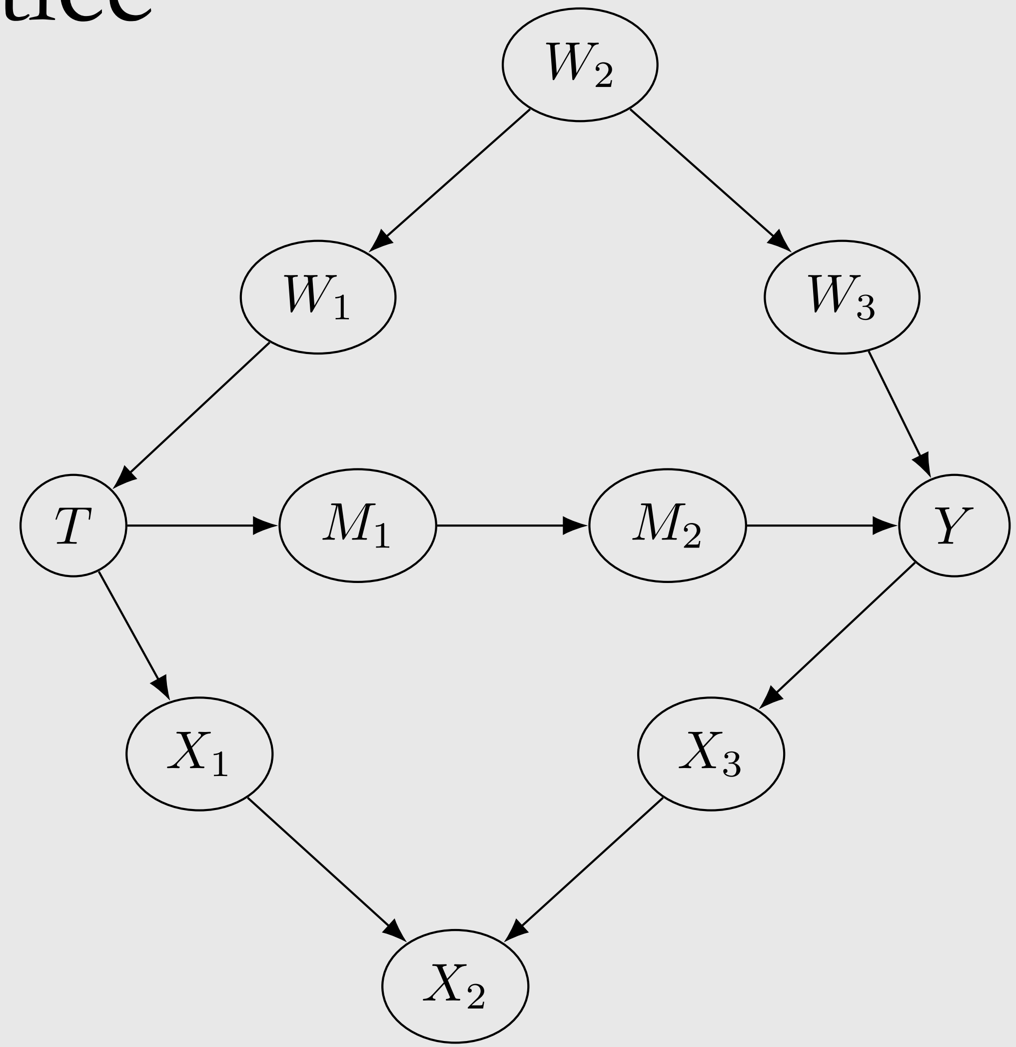
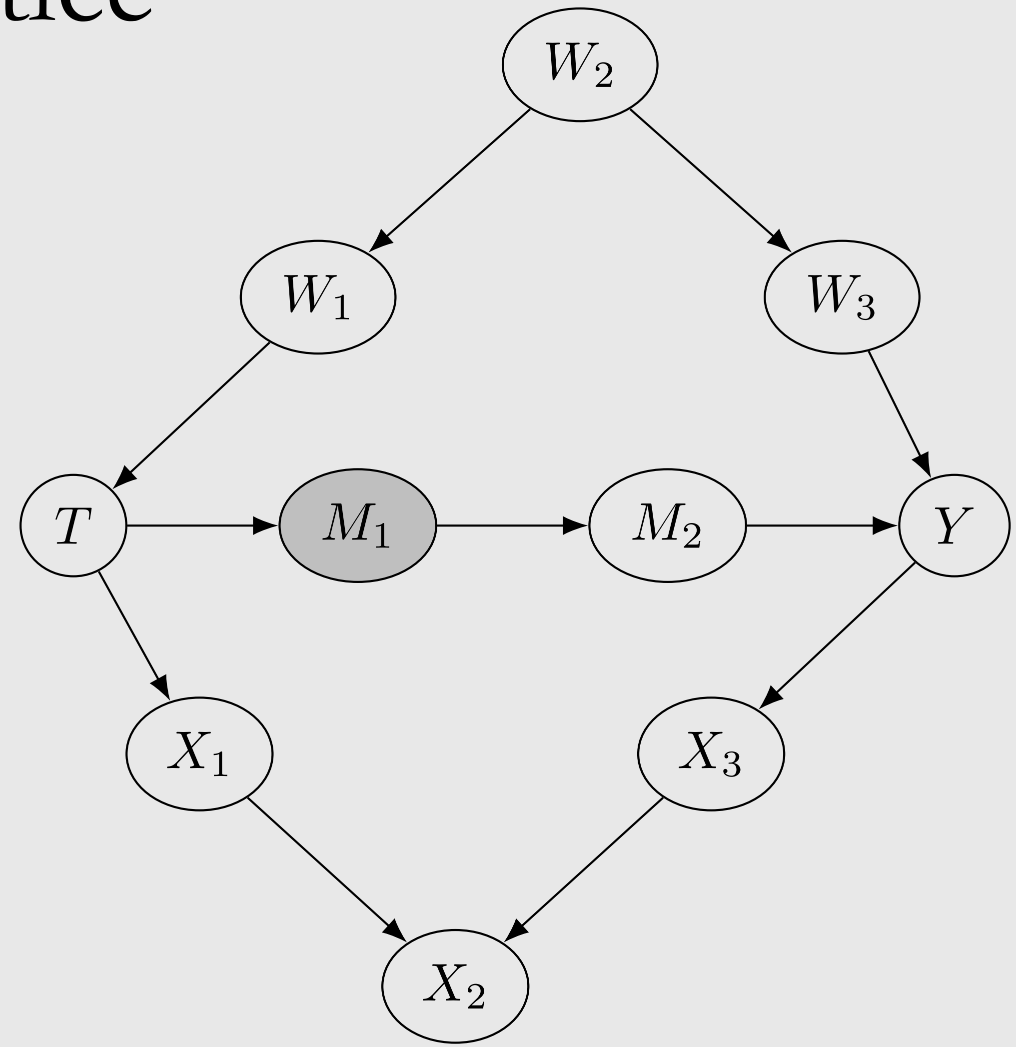
Markov assumption

d-separation practice



d-separation practice



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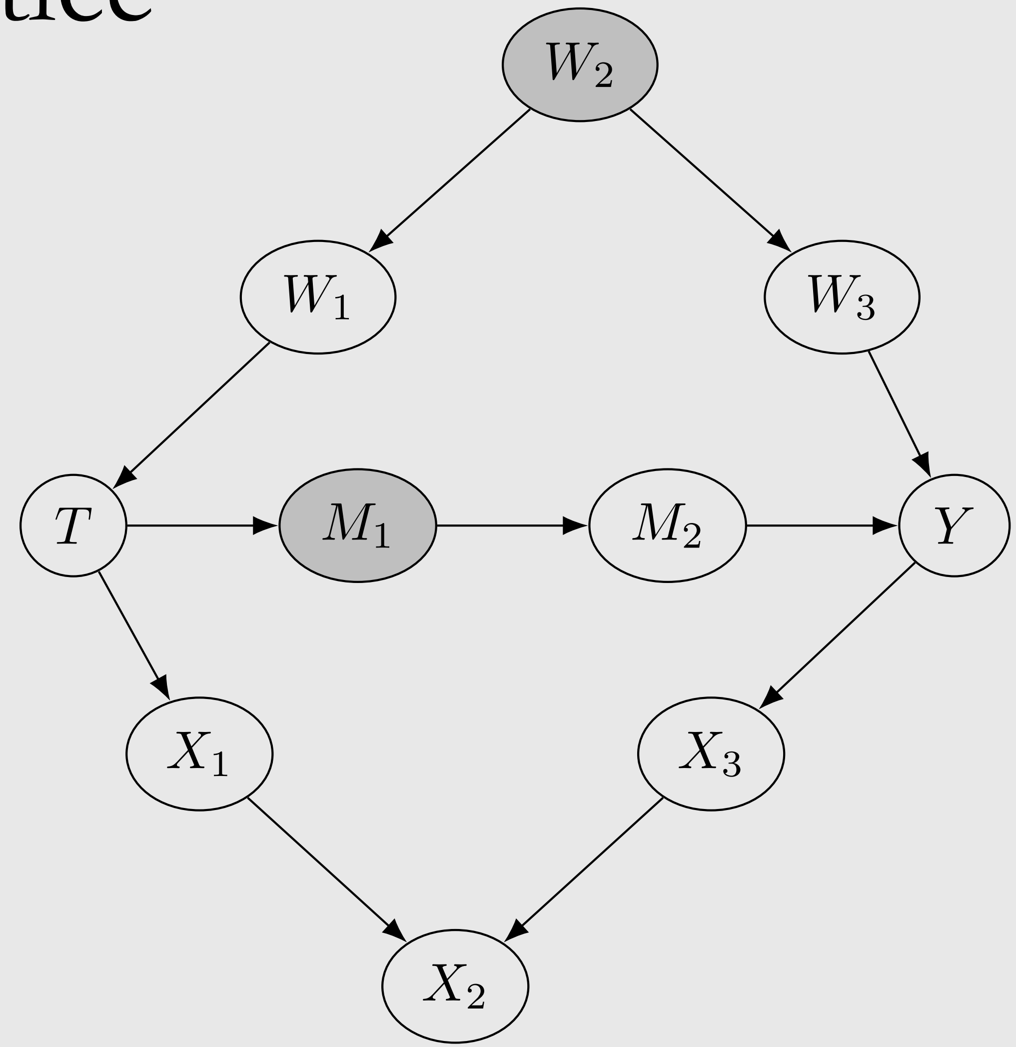

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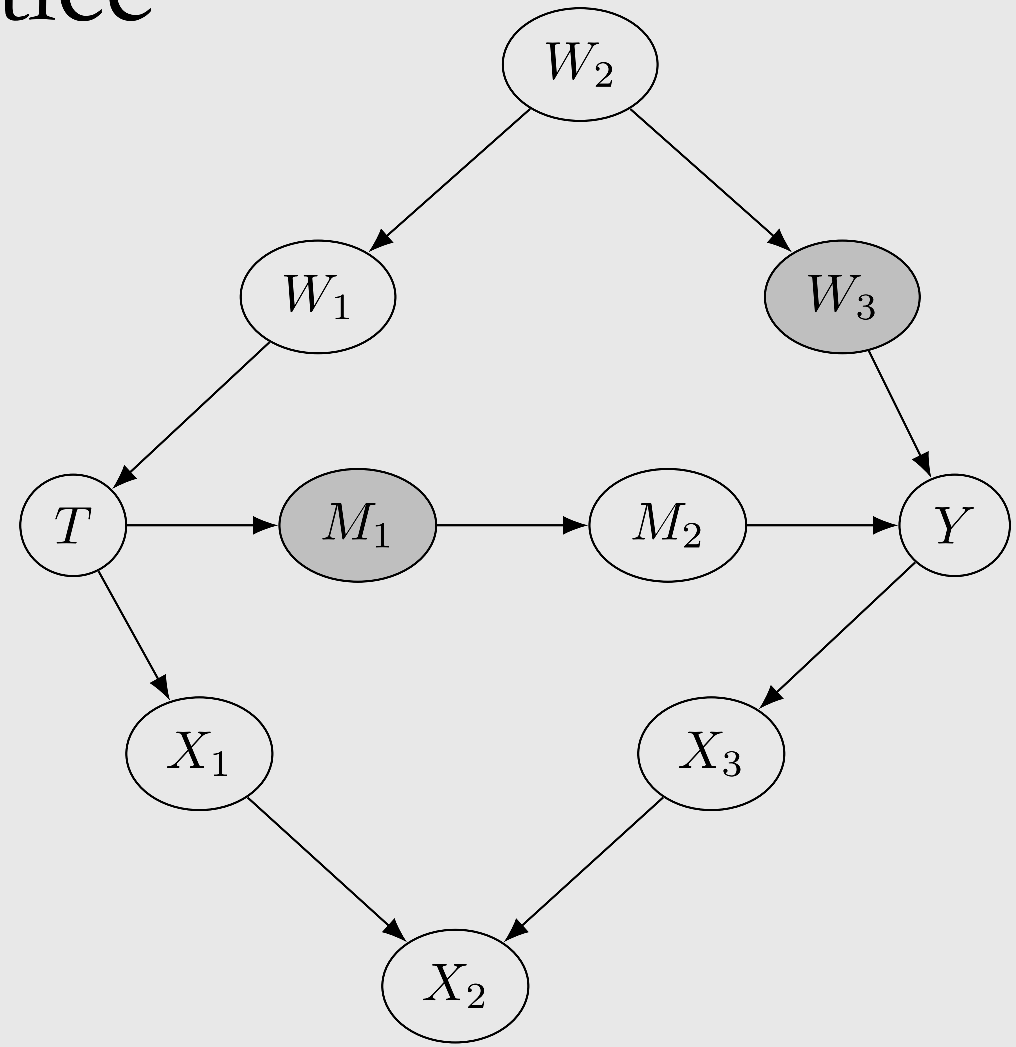

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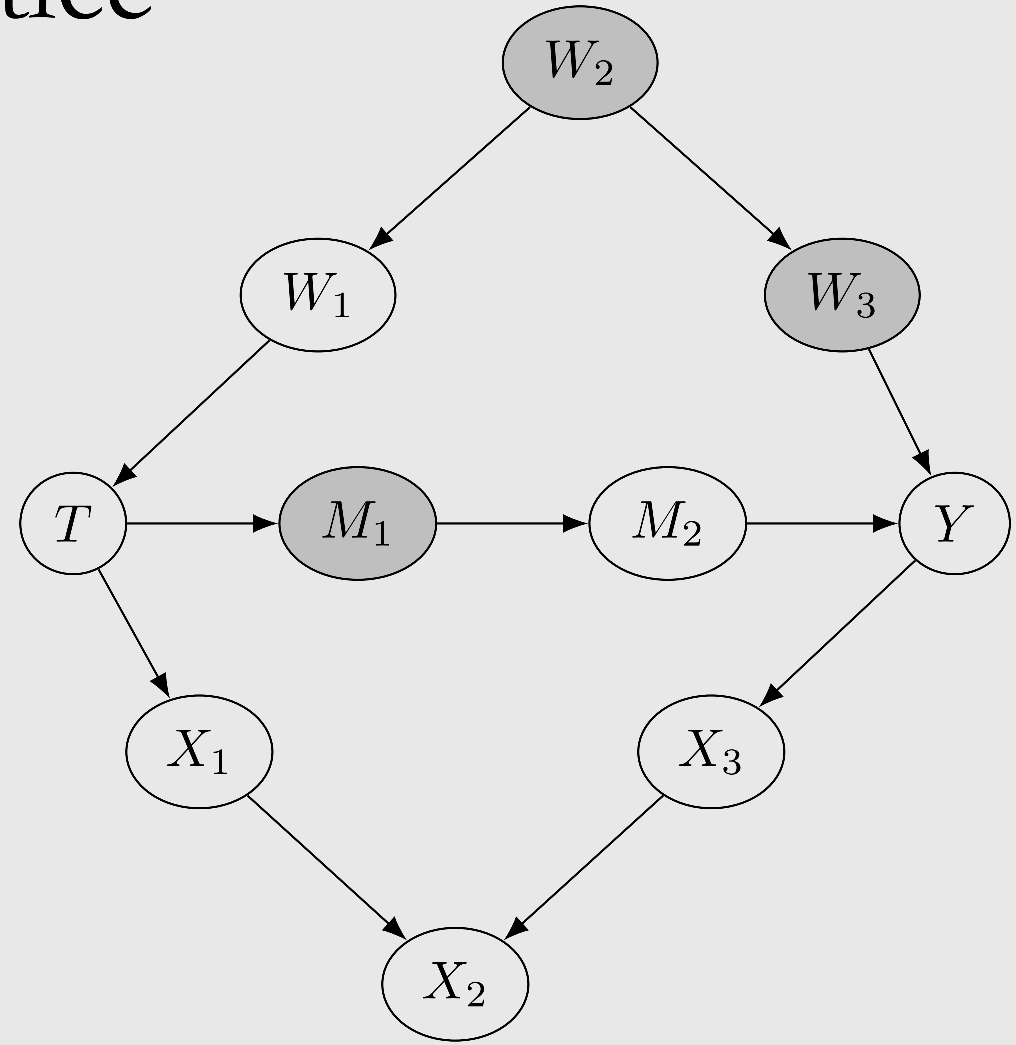

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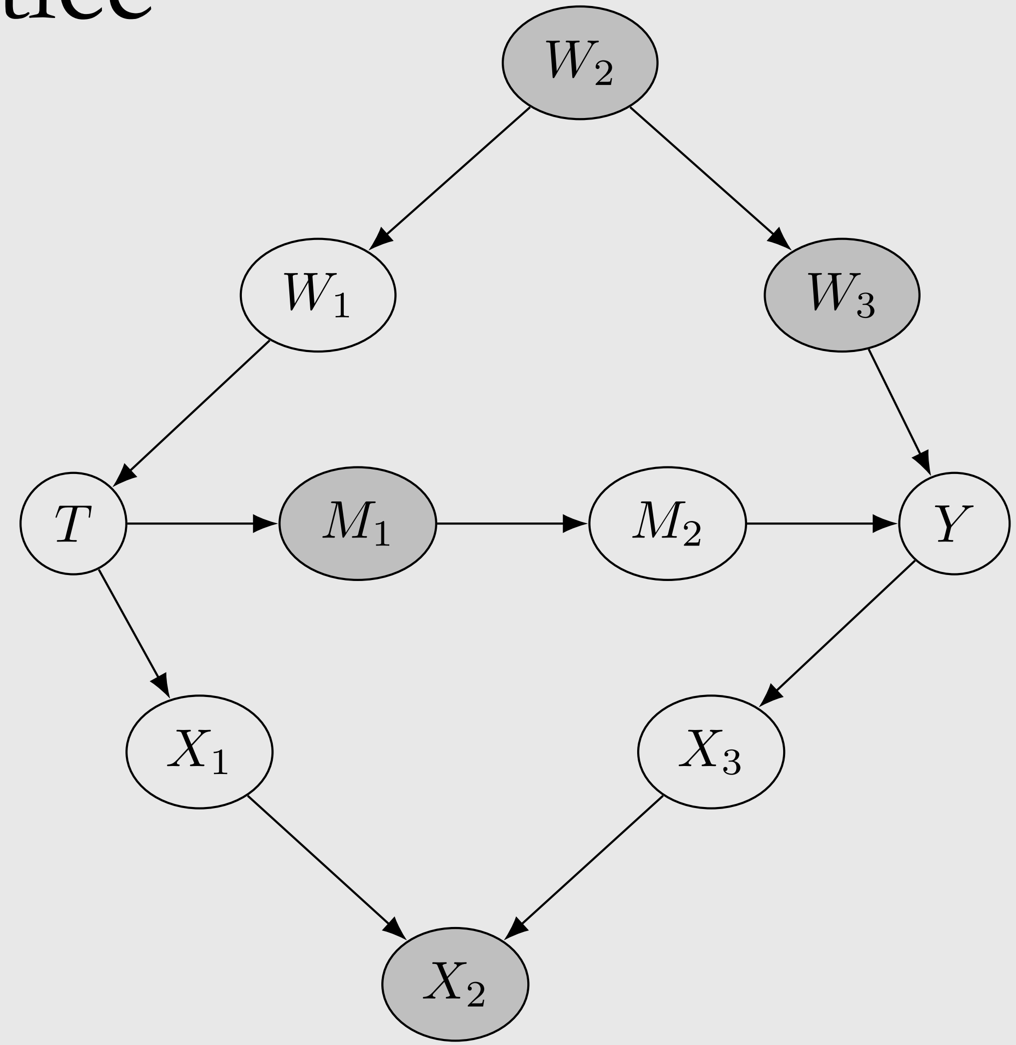

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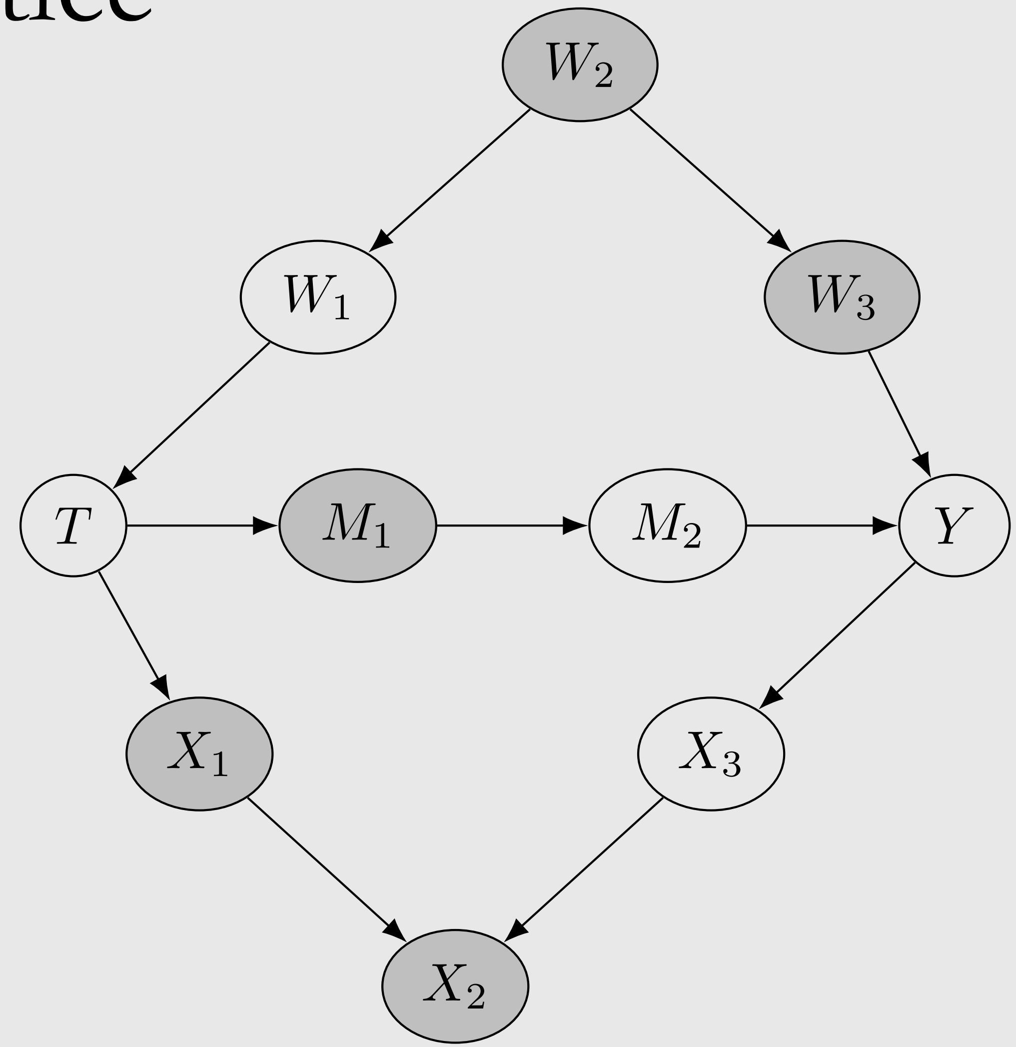

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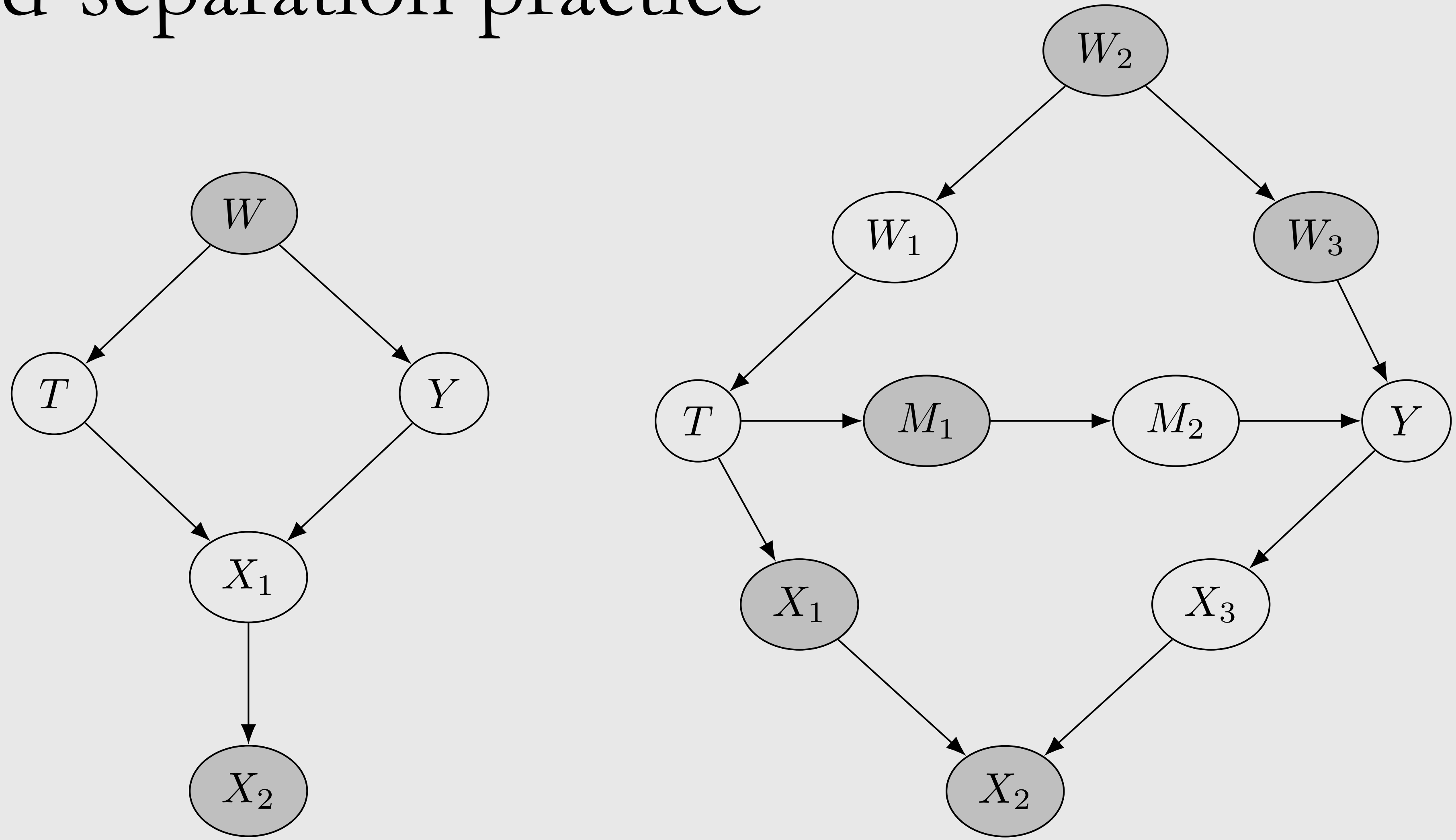

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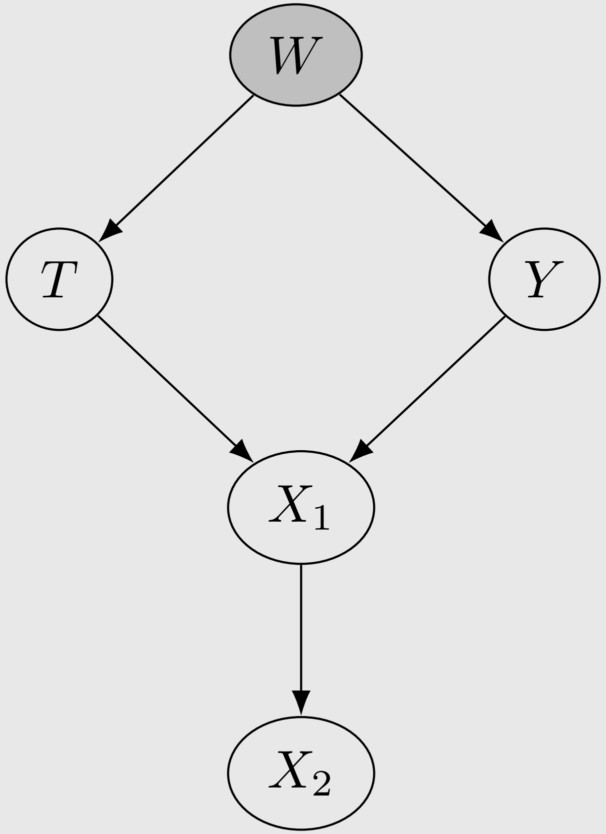
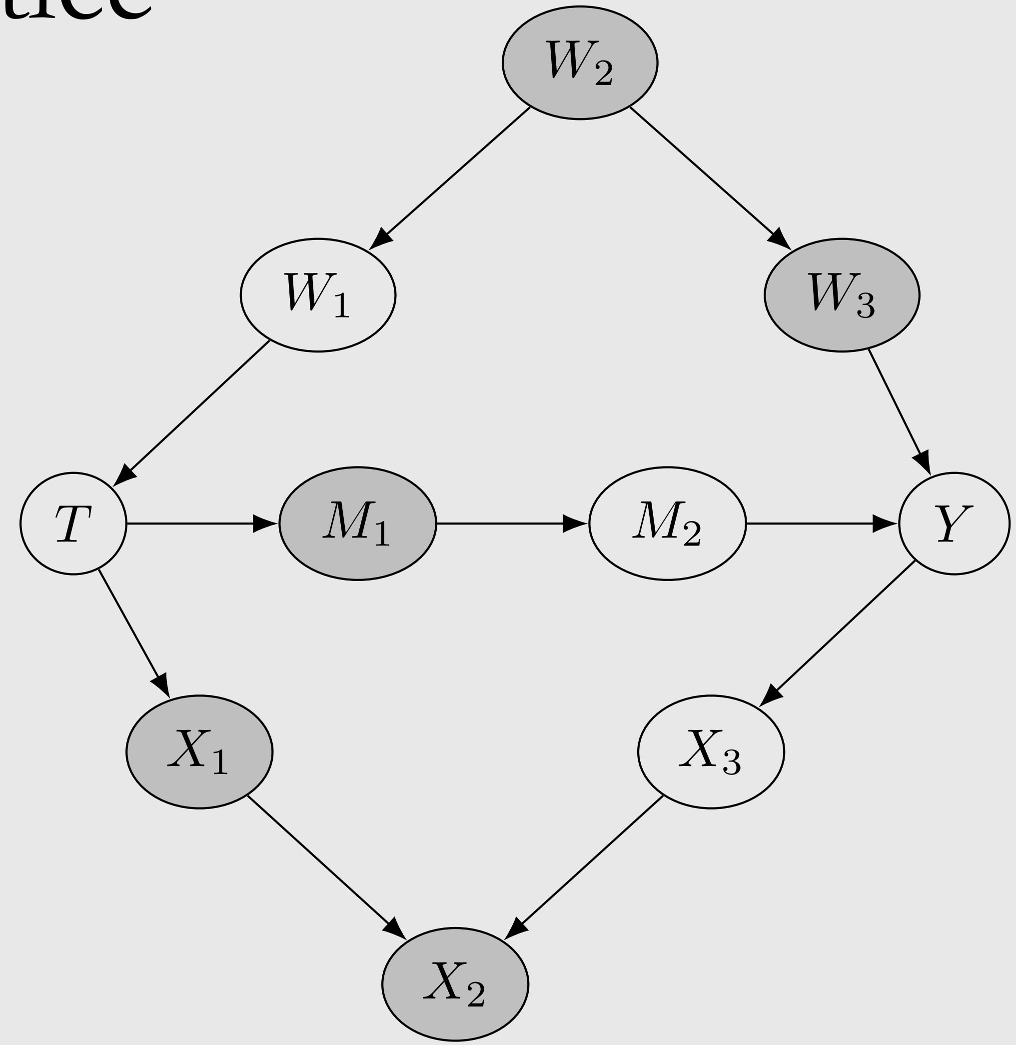
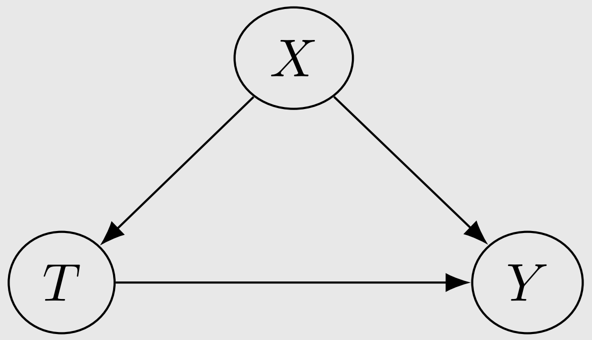




The flow of association and causation



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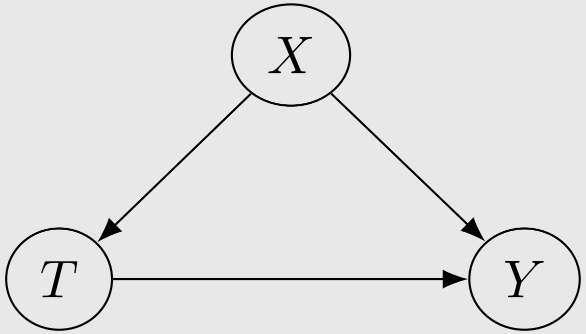

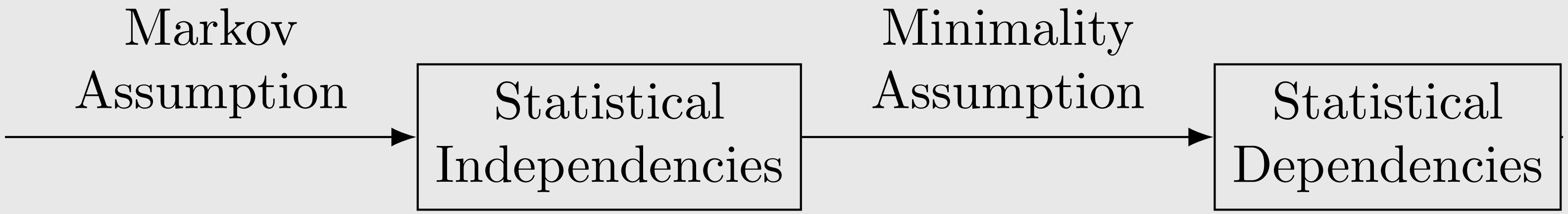

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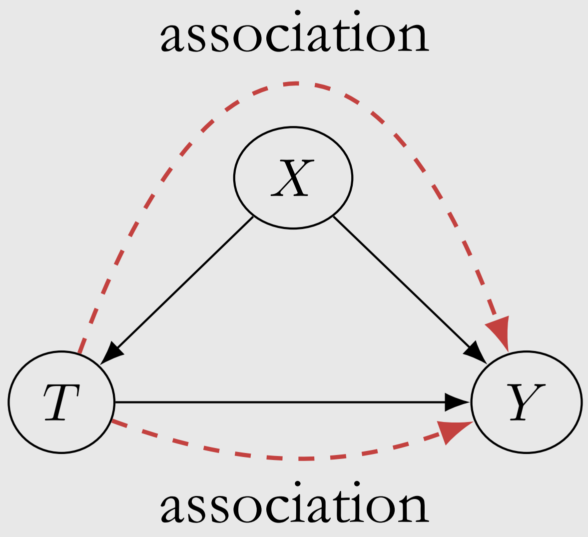

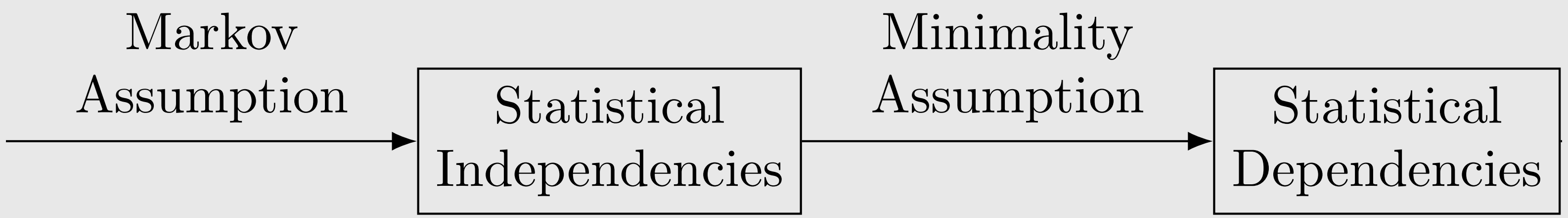

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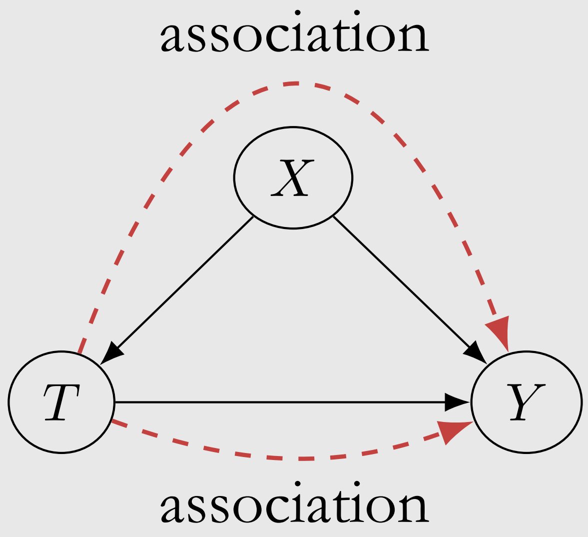

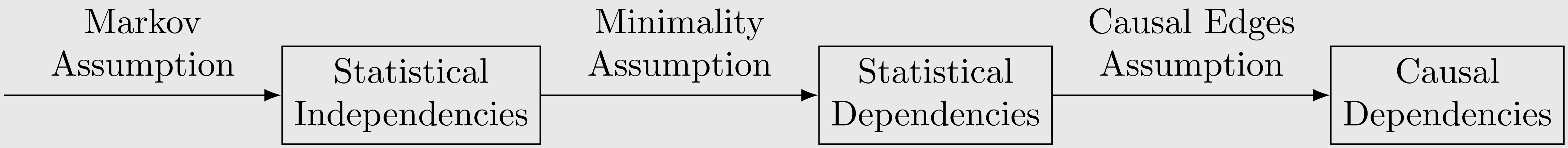

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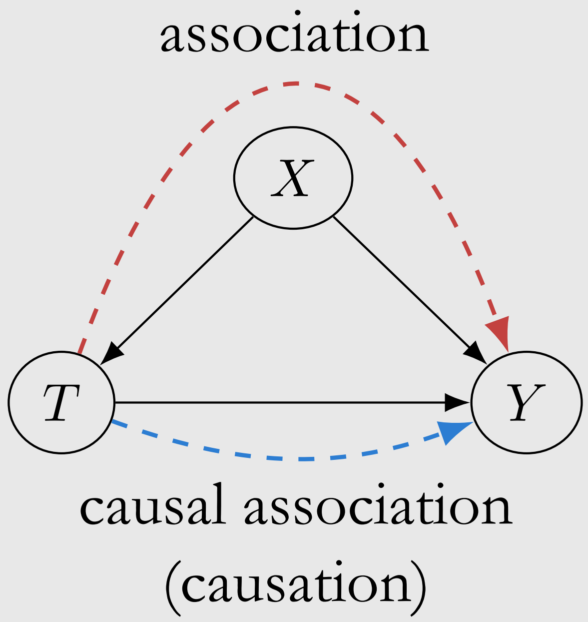

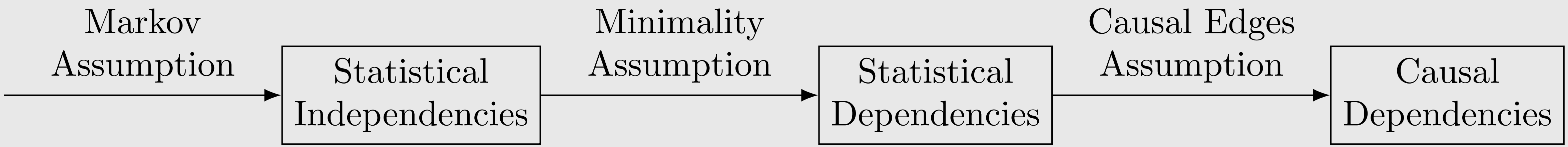

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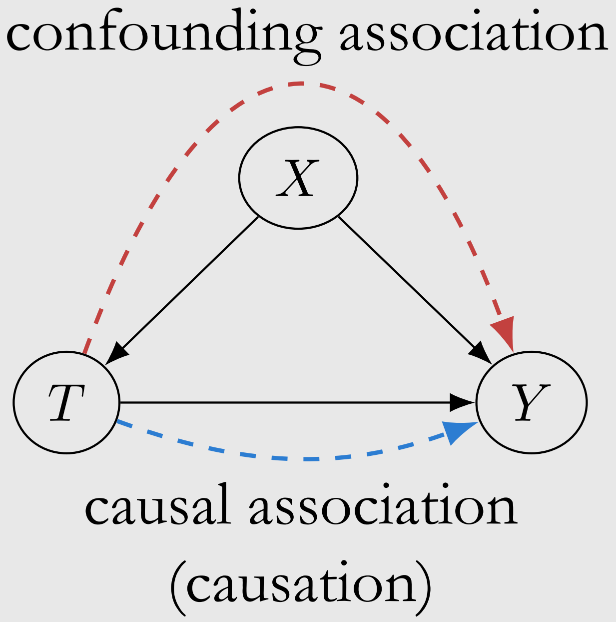
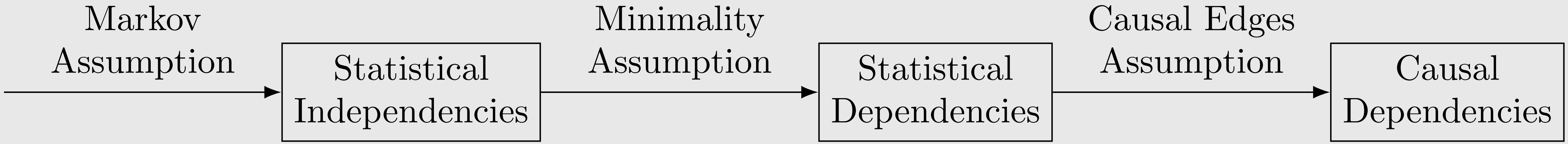
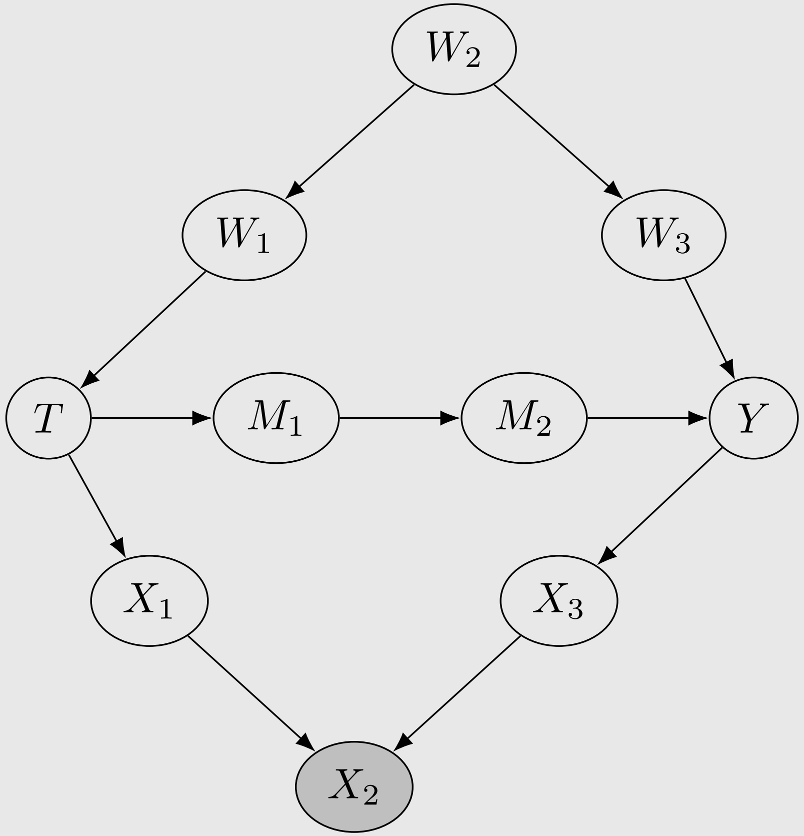
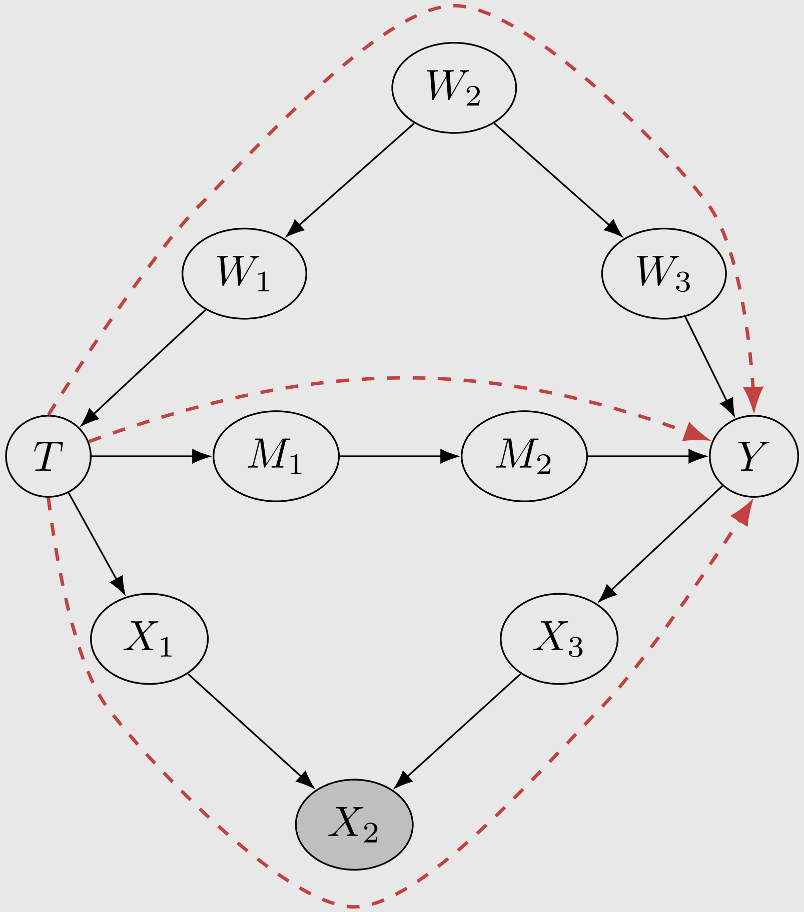





W_2
W_1
W_3
T
M_1
M_2
Y
X_1
X_3
X_2


W_2
W_1
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T
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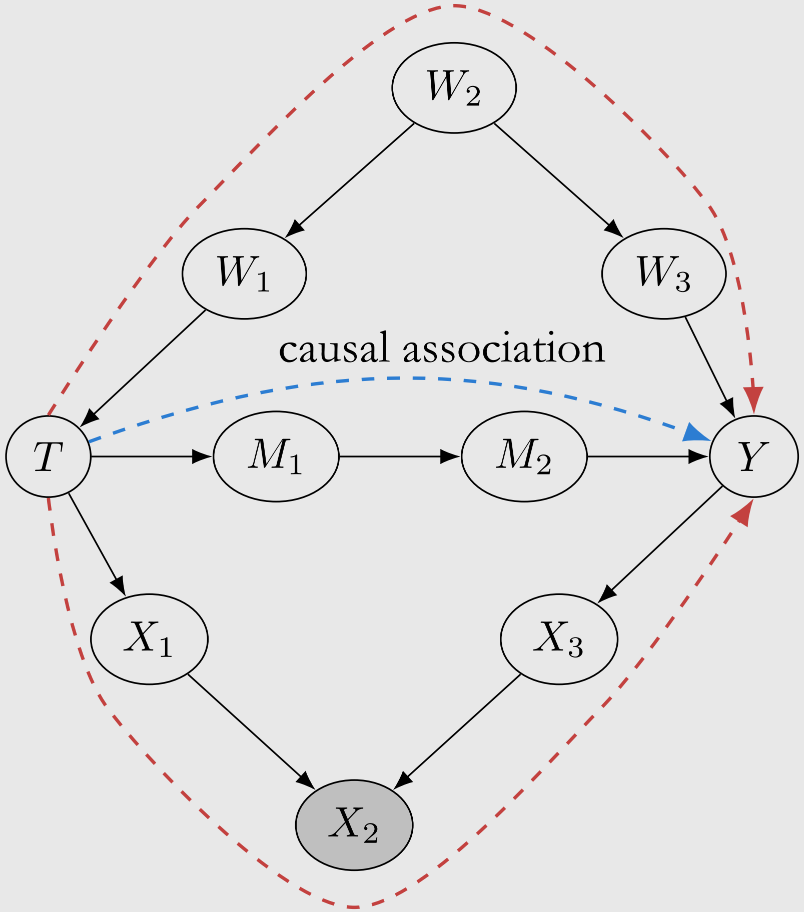
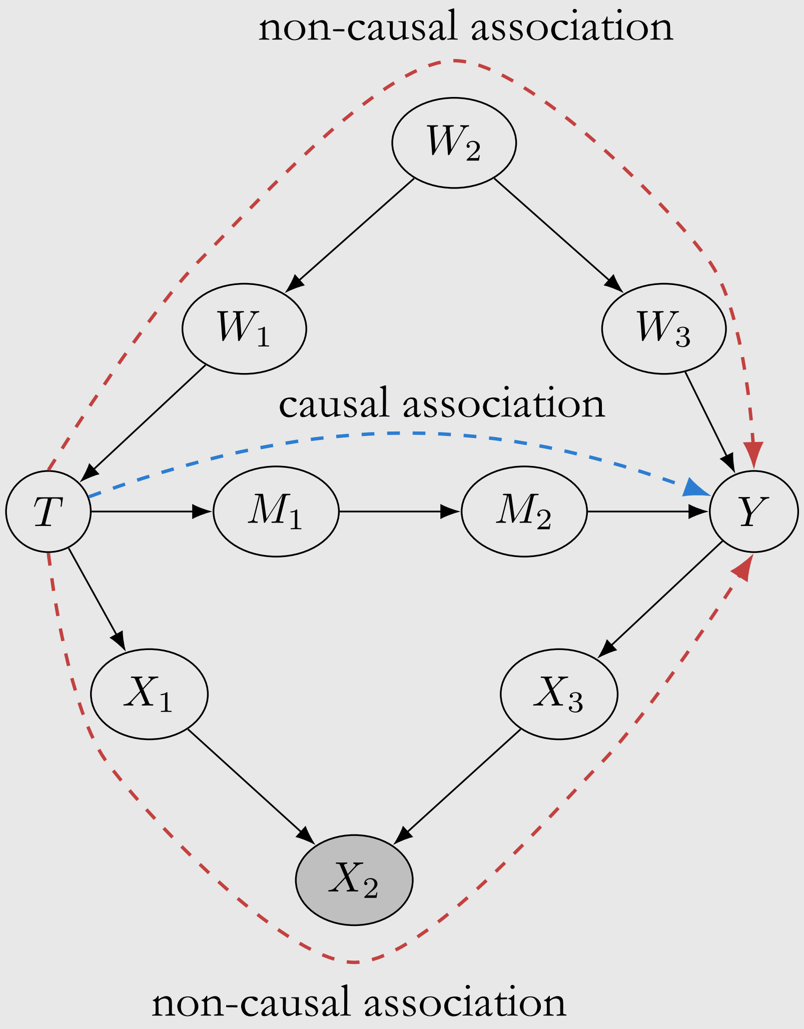


W_2
W_1
W_3
causal association
T
M_1
M_2
Y
X_1
X_3
X_2


non-causal association
W2
W1
W3
causal association
T
M1
M2
Y
X1
X3
X2
non-causal association